**MATHEMATICS II**

**UNIT-1 ORDINARY DIFFERENTIAL EQUATIONS**

1. **What is Inverse operator?**

**ANSWER:**  Inverse operator

 (i)  (ii)   

 (iii)  (iv) 

1. **What is meant by Singular solution?**

**ANSWER:**  The Solution of a differential equation which is not obtained from the general solution is known as **Singular solution.**

1. **What is Exponential shift rule?**

 **ANSWER:**  Let X=, , where  is of the form sincosor 

 Particular integral = 

This rule we call it as **Exponential shift rule.**

**4.** **Solve **

**ANSWER:** Given ****

The Auxillary equation is ****

i.e., ****

i.e., ****

****Complementary function =****

**The General solution is given by **

1. **Solve **

**ANSWER:** Given ****

The Auxiliary equation is ****

i.e.,  ⇒=1(thrice)

****Complementary function =

**Hence the solution is **

1. **Solve** 

**ANSWER:** Given 

The Auxiliary equation is

 i.e., 

 i.e.,

 i.e.

****Complementary function =

**Hence the solution is **

1. **Find the Complementry Function of **

**ANSWER:** Given ****

The Auxiliary equation is ****

i.e., 

 i.e., 

 i.e., 

****Complementary function =

  **C.F =**

**8. Solve** 

**ANSWER:** Given 

The Auxiliary equation is

 i.e., 

 i.e., 

****Complementary function =

**Hence the solution is **

**9. Solve** 

**ANSWER:** Given 

 i.e., 

The Auxiliary equation is 

 i.e., 

 i.e., 

****Complementary function =

**Hence the solution is**

1. **Solve **

**ANSWER:** Given ****

The Auxiliary equation is ****

i.e., 

 i.e., 

 i.e., (twice)

****Complementary function =

 **The Complete solution is **

**11. Solve** 

**ANSWER:** Given 

The Auxiliary equation is

 i.e., =2(twice)

****Complementary function =

Particular Integral =  [Replace D by 2]

 = [Ordinary rule fails]

 = [Ordinary rule fails]

 =

 = 

**The Complete solutuion +**

**12. Find the Particular Integral of **

**ANSWER:** Given 

 i.e., 

Particular Integral = 

 =

 =

 = [Replace D by 2]

 =

 =

 ** The Particular solution is =**

**13. Find the particular integral of **

**ANSWER:** Given 

 i.e., 

Particular Integral = y­p=  [Replace D2 by -32]

 =

 =

 =

 = [Replace D2 by -32]

 =

 =

 =

** The Particular solution is = **

**14. Find the particular integral of **

**ANSWER:** Given ****

 i.e., 

Particular Integral = y­p=  [Replace D2 by -22]

 =

 = [Ordinary rule fails]

 =

** The Particular solution is =**

**15**. **Find the particular integral of **

**ANSWER:** Given ****

Particular Integral = y­p= 

 =

 =

 =

 =

** The Particular solution is =** 

16. **Find the particular integral of **

**ANSWER:** Given ****

Particular Integral = y­p= 

 =

 =$e^{x}\frac{1}{D^{2}}sinx$

 = $e^{x}\frac{1}{-1}sinx$

 =$-e^{x}sinx$

** The Particular solution is =** $-e^{x}sinx$

17. **Find the particular integral of**$ \left(D^{2}+4D+4\right)y=\frac{e^{-2x}}{x^{2}}$

**ANSWER:** Given $\left(D^{2}+4D+4\right)y=\frac{e^{-2x}}{x^{2}}$

Particular Integral = y­p= $\frac{1}{ \left(D^{2}+4D+4\right)}\frac{e^{-2x}}{x^{2}}$

 = $\frac{1}{ \left(D+2\right)^{2}}\frac{e^{-2x}}{x^{2}}$

 = $e^{-2x}\frac{1}{ D^{2}}\left(\frac{1}{x^{2}}\right)$

 = $e^{-2x}\frac{1}{ D^{2}}\left(x^{-2}\right)$

 = $e^{-2x}\left(-logx\right)$ $\left[Since \frac{d^{2}\left(-logx\right)}{dx^{2}}=\frac{1}{x^{2}}\right]$

 = $-e^{-2x}logx$

** The Particular solution is =** $-e^{-2x}logx$

**18. Find the particular integral of**$\left(D^{3}-3D^{2}+3D-1\right)y=x^{2}e^{x}$

**ANSWER:** Given $\left(D^{3}-3D^{2}+3D-1\right)y=x^{2}e^{x}$

Particular Integral = y­p= $\frac{1}{\left(D^{3}-3D^{2}+3D-1\right)}x^{2}e^{x}$

 = $\frac{1}{\left(D-1\right)^{3}}x^{2}e^{x}$

 = $e^{x}\frac{1}{\left(D+1-1\right)^{3}}x^{2}$ **[**Replace D by D+1**]**

 **=** $e^{x}\frac{1}{\left(D\right)^{3}}x^{2}$

 = $e^{x}\frac{x^{5}}{60}$  **[**Integrating $x^{2}$three times**]**

** The Particular solution is =** $e^{x}\frac{x^{5}}{60}$

**19. Solve** $\frac{d^{2}y}{dx^{2}}-3\frac{dy}{dx}+2y=1+x$

**ANSWER:** Given $\frac{d^{2}y}{dx^{2}}-3\frac{dy}{dx}+2y=1+x$

 i.e $\left(D^{2}-3D+2\right)y=1+x$

The Auxiliary equation is $m^{2}-3m+2=0$

 $⇒\left(m-1\right)\left(m-2\right)=0$

 $⇒m=1,2$

**Complementary function =**$Ae^{x}+Be^{2x}$

Particular Integral = y­p= $\frac{1}{\left(D^{2}-3D+2\right)}(1+x)$

 **=** $\frac{1}{2}\frac{1}{\left(\frac{D^{2}-3D}{2}+1\right)}(1+x)$

 = $\frac{1}{2}\left[1+\left(\frac{D^{2}-3D}{2}\right)\right]^{-1}(1+x)$

 = $\frac{1}{2}\left\{1-\left(\frac{D^{2}-3D}{2}\right)+…\right\}(1+x)$

 = $\frac{1}{2}(1+\frac{3D}{2})(1+x)$

 = $\frac{1}{2}\left\{1+x+\frac{3}{2}\right\}$

 **=** $\frac{1}{2}\left\{x+\frac{5}{2}\right\}$

 **=**$\left\{ \frac{x}{2}+\frac{5}{4}\right\}$

** The Particular solution is =** $\left\{ \frac{x}{2}+\frac{5}{4}\right\}$

**The Complete solution **$Ae^{x}+Be^{2x}+ \frac{x}{2}+\frac{5}{4}$

**20. Solve** $ x\frac{d^{2}y}{dx^{2}}+\frac{dy}{dx}=0$

**ANSWER:** Given $x\frac{d^{2}y}{dx^{2}}+\frac{dy}{dx}=0$

 i.e. $\left[xD^{2}+D\right]y=0$

 ⇒ $\left[x^{2}D^{2}+xD\right]y=0$ [Multiplying by x]

 Put $x=e^{z}$

 l$ogx=z$

 $xD=D'$

 $x^{2}D^{2}=D^{'}(D^{'}-1)$

 $\left[D^{'}\left(D^{'}-1\right)+D^{'}\right]y=0$

 $[D^{'}^{2}-D^{'}+D']y=0$

 $ D'^{2}y=0$

Auxillary Equation is $ m^{2}=0$

 $m=0,0$

Complementry Function is y = $[Ax+B]e^{0x}$

**The Complete Solution is y=**$ Ax+B$

**PART - B**

1. Solve the equation (i)(D2 – 4D + 3)y = sin3x + x2,(ii) (D2 + 4)y = x4 + cos2x.
2. Solve the equation (i) (D3 + 8)y = x4 + 2x + 1 + cosh2x, (ii) (D4 – 2D3 + D2)y = x2 + ex.
3. Solve the equation (i)(D2 + 1)2y = x4 + 2sinx cos3x, (ii) (D3 – 3D2 + 3D – 1)y = e-xx3.
4. Solve the equation (i) (D4 – 4)y = x2cosh2x,(ii) (D4 – 2D2 + 1)y = (x + 1)e2x.
5. Solve the equation (i) (D2 + 5D + 4)y = e-xsin2x,(ii) (D4 – 1)y = cos2x coshx.
6. Solve the equation (i)(D2 – 4D + 13)y = e2x cos3x,(ii) (D2 + D + 1)y = e-x sin2(x/2).
7. Solve the equation (i) (D2 + 2D + 5)y = ex cos3x, (ii) (D2 + 4D + 8)y = 12e-2x sinx sin2x.
8. Solve the equation (i) (D3 – 1)y = x sinx,(ii) (D2 + 4)y = x2 cos2x.
9. Solve the equation (i)(D2 – 4D + 4)y = 8x2 e2x sin2x,(ii)(D2 + a2)y = secax.
10. Solve the equation (i) (D2 – 4D + 3)y = sin3x cos2x,(ii)(D4 – 1)y = cosx coshx.
11. Solve (i)(x2D2 + xD + 1)y = logx sin(logx),(ii)(x2D2 +4xD + 2)y = sinx.
12. Solve (i)(x2D2 + 2xD – 20)y = (x2 + 1)2,(ii) (x4D3 +- x3D2 + x2D)y = 1.
13. Solve (i) [(3x + 2)2D2 + 3(3x + 2)D – 36]y = 3x2 + 4x + 1,

 (ii) [(x + 1)2D2 + (x + 1)D + 1]y =4 cos log(x + 1).

1. Solve the simultaneous equations (dx/dt) + 2x – 3y = 5t,(dy/dt) – 3x + 2y = 2e2t.
2. Solve the simultaneous equations Dx – (D – 2)y = cos2t,(D – 2)x + Dy = sin2t.

**Unit – II Vector Calculus**

1. If  (x,y,z) is a constant , then P.T .

Ans :

 If  (x,y,z) is a constant , its partial derivatives with respect to x,y,z will be zero .ie.,

 

 

1. Find .

Soln:

 We’ve 



 

 

 

 =

 = =****

1. P.T 

Soln :

 

 =i =

 =

 =

1. Find 

Soln:

 

 =

 =

 =

1. P.T 

Soln:

 r

 r

 

 = 

 

 

 

 

1. Find the directional derivative of  at (1,2,3) in the direction of 

Soln:

 

 D.D at (1,2,3) in the direction of 

 

 

 D.D of  at (1,2,3)is the direction of  

 

1. Find the angle between the surfaces xlogz=y2-1 and x2y=2-z at the point (1,1,1)

Soln. 

 

 Angle between  and  is

 at (1,1,1) at(1,1,1)

 

 

 

 

 

 

 

 

 

 

1. Find the unit normal to the surface s2+y2-z=10 at (1,1,1).

Soln. Unit Normal  at (1,1,1) =

 

 

 

 

 

 

1. If . Find div 

Soln. div curl ------------(1)

 

 

 div 

1. If  is solenodial. Find the value of λ.

Soln. Given is solenodial 

 

 =0

 

 

1. If , Evaluate where c is the curve in the xy-plane y=2x2 from (0,0) to (1,2)

Soln. 

  (0,0) to (1,2)

   

 

 

 

1. If . Check whether the integral is independent of the path c

Soln. T.P.T ,  is independent of the path c, we’ve to prove that 

 

 

 is independent of the path.

1. P.T. 

Soln. 

 

1. Find the Constants a,b,c so that  is irrational

Soln. Given, 

 

 c+1=0 4-a=0 b-2=0

 c=-1 a=4 b=2

1. If s is any closed surface enclosing a volume v and is position vector of a point. P.T. 

Soln. Gauss Div Theorem

 

 =1+1+1=3

 

1. P.T 

Soln. 

 

 

 

1. Find the unit normal vector to the surface xy3z2=4 at(-1,-1,2)

Soln.   

1. Apply Greens’ theorem at plane P.T. the area enclosed by a simple closed curve c is 

Soln. Green’s Theorem

 

  = 2(Area of R)

 Area of R 

1. If and are irrotational, P.T is solenoidal

Soln. T.P. 

 W.K.T, 

  and  are irrotational  & 

 

  is solenodial

1. If Find the 

Soln.

 

 

  ****

1. If  evaluate along the line y=x from (0,0) to (1,10

Soln. 

 

  (0,0) (1,1)

 dy=dx x=0 1

 y=0 1

 

1. Find the directional derivative of xyz at (2,1,10 in the direction 

Soln. Directional derivative in the direction of

 

  at (2,1,1), 

 

 

 D.D in the direction of 

 

 

1. State green’s Theorem:

 (or)

 State relationship between line integral and a double integral

Soln. If c is a simple close curve enclosing a region R in the xy-plane and p(x,y), Q(x,y) and its first order partial derivates are continuous in R, then

  Where c is described in the anticlockwise direction.

1. State Stoke’s theorem

 (or)

 Relationship between line and surface integral

Soln. If s is an open two sided surface bounded by simple closed curve cand if is a vector point function with continuous first order partial derivatives on s, then



Where c is described in the anti-clockwise direction as seen from the positive tip of the outward drawn normal at any point of the surface S.

1. State Gauss Divergence theorem

Soln. If s is a close surface enclosing a region of space with volume v and if is a vector point function with constants first order partial derivates in v, then

 

**PART - B**

1. Evaluate using Green’s theorem in the plane for ∫ (xy + y2) dx + x 2 dy where C is the closed curve of the region bounded by y = x and y = x2.
2. Evaluate ∫ [ (sinx –y) dx – cos xdy] where c is the triangle with vertices (0,0), (π/2 ,0) and (π/2,1).
3. Verify Green’s theorem in the plane for ∫(3x2 -8y2)dx + (4y-6xy)dy where c is the boundary of the region defined by x = 0, y = 0, x+y =1.
4. Verify Green’ s theorem in plane for ∫ x2(1+y)dx + (y3 +x3) dy where C is the square bounded by x = a, y = a.
5. Verify Divergence theorem for F = (x2-yz) i + (y2 –zx) j + (z2 – xy) k taken over the rectangular parallelepiped 0≤ x ≤ a, 0≤ y ≤ b , 0≤ z ≤ c.
6. Verify Divergence theorem for F = (x2-yz) i + (y2 –zx) j + (z2 – xy) k taken over the rectangular parallelepiped 0≤ x ≤ 3, 0≤ y ≤ 3 , 0≤ z ≤ 3.
7. Verify Divergence theorem for F = (2x –z) i + x2 y j – xz2 k over cube bounded by x = 0, x =1, y =0 , y=1, z = 0 and z =1.
8. Verify Divergence theorem for F = (2x –z) i + x2 y j – xz2 k over cube bounded by x = 0, x =2, y =0 , y=2, z = 0 and z =2.
9. Verify Divergence theorem for the function F = 4xi – 2 y2 j + z2 k taken over the surface region bounded by the cylinder x2+y2 = 4 and z =0, z =3.
10. Verify Stoke ‘s theorem for a vector field F = (x2 –y2) i + 2xy j in the rectangular region of the xoy plane bounded by the lines x = -a , x = a, y = 0 and y =b.
11. Verify stoke’s theorem for F = (y-z+2) i +(yz + 4) j – xz k where S is the surface of the cube x =0 , x=2 , y =0 , y= 2, z =0 and z = 2 above the xy- plane.
12. Verify stoke’s theorem for F = y2 i + y j –xz k over the upper half of the sphere x2+y2+z2 = a2 , z ≥ 0.

**Unit – III Analytic Functions**

1. If *u* and *v* are harmonic then can we say *u + iv* as analytic?

**Ans**. Yes. But it is not always. Eg., *u = x, v = -y.*

1. State the sufficient conditions for the function F(z) to be analytic

**Ans**. The single valued continuous function w=*f*(z)=*u+iv* is analytic in a region R if the four partial derivatives  exist, are continuous and satisfy CR equation at each point in R,

1. Define an enter function?

**Ans**. A function *F(z)* which is analytic at every point of the complex plane is called as entire function. Eg.

1. state the Cauchy –Riemann – in polar co-ordinates .

**Ans**.  

1. Give an Example that the C-R equation are necessary but not sufficient for a function to be analytic?

**Ans**. , When 

 =0, when z = 0

1. If *u+iv* is analytic, show that *v-iu* is also analytic

**Ans**. Given: *f(z) = u+iv* is analytic

 Hence it satisfies C-R Equation (i.e) *Ux=Vy*  ➀

 *Uy=-Vx* ➁

*g(z) = v-iu*, replace u by v & v by –u in ➀,➁ *Ux=Vy, Uy=-Vx*

1. When *f(z)=u+iv* is analytic only if *Ux=Vy* and *Uy*
2. If *f(x)=u+iv* is an analytic function. If  find *f(z)*

**Ans**. Let  ➀

 Diff ➀ partically w.r to x  ➁

 Diff ➀ partically w.r to y 

 By C-R Equations  ➂

 

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 We know 

 By Milne Thomson Method, 

 

  

 

 

 *u+iv = excosy+iexsiny+c*

1. For what values of a,b,c the function f(z)=x+x+2ay+i(bx-cy) is analytic?

**Ans**. If f(z) = x – 2ay + i(bx-cy) is analytic

 Then it satisfies C-R equations ux=1, Uy=-2a

 V2=b, Vy=-c

  

 

 Hence for *c=-1* and any values of a and b satisfying *b=2a* then function f(z) is analytic.

1. If *u+iv* analytic, then the curves *u*=constant and *v*=constant are orthogonal.
2. If Orthogonal f is a fun of z alone then f is analytic whether it is true or false?

**Ans**. True

1. Choose the correct answer.

*W=f(z)* is analytic function of *z* then

 *   *

**Ans**. **

1. If *u* and *v* are harmonic then *u+iv* is analytic function?

**Ans**. True

1. Note: If *f(z)* is analytic function, then *kf(x)* is also analytic function
2. If *f(z)* and *g(z)* are analytic function *f(z)*+*g(z)* is also analytic.
3. verify whether  is analytic or not

**Ans**. 

  is not alnalytic

1. is anal;ytic everywhere except

a)z=1 b)z=±1 c)z=±i d)z=i

**Ans**. *z=±i* because *f(z)* becomes infinity at this point.

1. The Function is not analytic at *z=2i, -2i*
2. If *f(z)* is an analytic function of z*.* Prove that 

**Ans**. We know that 

 

 

1. State he two properties of analytic functions
	1. Both Real Part and imaginary part of an analytic function satisfy Laplace equation
	2. if *w=u+iv* is an analytic function then the curves of the family *u(x,y)*=Constant, cut orthogonally the curves of the family *v(x,y)*=constant=*c2*
2. State the necessary **Ans** sufficient condition for *f(z)* to be analytic.

**Ans**. Necessary condition: Let *w=f(z)=u+iv*  be an analytic function then  and  Sufficient condition: The single valued continuous function *u+iv* is an analytic in a region R if the four partial derivatives exists and continuous and satisfy the C-r Equation at each point in R.

1. Can U=3x2y-y3 be the real part of an analytic function

**Ans**. u=3x2y-y3

U*x*=6*xy*. U*xx* = 6*y*

U*y=*3*x*2*-*3*y*2 U*yy*=-6*y*

Uis harmonic and can be the real past of an analytic function.

1. State whether sin(*x+iy*) is analytic or not?



 

  

 Ux=ry and vy=-vx

 

1. Show that the function x is harmonic.

**Ans**: let 

  

 

 is harmonic.

1. Show that tanis harmonic

**Ans**:

 

 is harmonic .

1. Verify whether  is harmonic.

**Ans**: Let 

  

 

 

 

1. Verify whether w=sin x cos h y +I cos x sin h y is analytic or not?

**Ans**:

  

  

  &  and all the partial

 W is analytic

1. If is analytic, if the value of p is

a) b)0 c)2 d)1 .

 **Ans**:

 P=2

Proof:  

  

  

 By using&we get

 

 

 

 

1. Define conformal mapping?

**Ans**:

 A mapping that preserves angles between any oriented curves both in magnitude and in sense is called as a conformal mapping.

1. The mapping w=z is not conformal at\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Ans**:

 

 

 2z=0 when z=0At z=0, It is conformal.

1. Find the points at which the transformation w=sin z is not conformal.

**Ans**:

 

1. Define Bilinear transformation ?(or) Mobius transformation

**Ans**:

 ,where a,b,c,d are complex constants ad-bc#0 is known as bilinear transformation.

1. Define isogonal transformation

**Ans**. If a transformation is such that the angle between two curves *c1* and *c2* intersection at *(x0,y0)* in z-plane is equal to the angle between their images intersecting at (*U0,V0*) in the w plane then the transformation is called isogonal transformation.

1. Write the cross ratio of the points Z1, Z2, Z3, Z4

**Ans**. Cross Ration 

1. Write the invariant point of transformation 

**Ans**. ;  

1. Find the image of |z-2i|=2 where 

**Ans**. 

 

 Now |z-2i|=2 menas |s+iy-2i| = 2

 

1. Find the Critical point of 

**Ans**. The critical point are obtained by  

 

Hence the critical points are 

1. Find the bilinear transformation that has i and –i as fixed points.

**Ans**. The Bilinear transformation is given by 

Since i and –i are the fixed points we have and 

 

 

The bilinear transformation is  where a and b are arbitary.

1. Show that the function f(z)=|z|2 is not analytic at any point

**Ans**. Let z=u(x,y)+iv(x,y)

 

 Hence *u(x,y)=x2+y2* *v(x,y)*=0

 The Cauchy Riemann equation are ux=vy uy= -vx

 2x=0 2y=0

Which is possible only when z=y =o. Thus the function is differentiated only at ongin.

1. Find the analytic function of f(z)=(x-y)2+2i(x+y).

**Ans**:



 

2(x-y)=2 -2(x-y)=-2

x-y=1 x-y=1

C.R equation are satisfied only if x-y=1.

1. Define a critical point of the bilinear transformation?

**Ans**:

 The point at which the mapping w=f (z) is not conformal (i,e) f (z) =0 is called a critical point of the mapping .

 The transformation w = f(z) is conformal ,z =f(w) is also confomal its critical point occurs at 

42. If f(z) = z analytic ? Justify.

**Ans**:

 The sufficient conditions for the function w =f (z) to be analytic in argion R are 1.ux ,uy ,vx and vy are existing to ux =vy and uy = -vx at every point of R.

In this example ,given : f (z) = z

 U+iw =(x+iy)3=x 3+3x2 (iy)+3x(iy)2+ (iy)3

= (x3-3xy2) +I (3x2y-y3)

Part b.

1. Evaluate c∫ 1/(z2+4) dz where c is |z|=3.
2. Obtain Taylor’s series to represent the function z2-1/(z+2)(z+3) in the region |z|<2.
3. Expand 1/ (z-1)(z-2) as a Laurent’s series valid in the regions 1<|z|<2 and |z|>2.
4. Find the residue o Z3/ (z-1)4(z-2)(z-3).
5. If c is the circle |z|=3, evaluate c∫tanz dz.
6. Evaluate c∫dz/ zsinz where c is |z|=1 using cauchy’s residue theorem.
7. Using contour integration evaluate 0∫2π dθ/(13+5sinθ).
8. Using contour integration evaluate 0∫2π cos2θ dθ/(5-4cosθ).
9. Using contour integration evaluate 0∫π dθ/(a2+sin2θ). a>0.
10. Using contour integration evaluate -∞∫∞ xdx/(x+1)(x2+1).
11. Using contour integration evaluate -∞∫∞ (x2-x+2) dx /(x4+10x2+9).
12. Prove that 0∫∞ log(1+x2) / (1+x2) dx = π log2

**Unit –IV-Complex Integration**

1. What is the Value of  if c is |x| = ½

Soln. 

 The pole is at z= -1 lies outside the circle |z| = ½

  = 0 [By Cauchy’s integral Theorem]

1. Find the residue at z = 0 of the function f(z) = 

Soln. [Reside of f(z) at z = 0]

 = 

= 

By using L's Hospital Rule

= 

= 

= 1

1. State Cauchy’s integral Formula

Soln. If f(z) is analytic within and on a simple closed curve c and z0 is any point

inside c then

 f(z0) = 

1. Find the Laurent’s Expansion of f(z) = in |z| > 1

Soln. |z| > 1 

  =  = 

 = 

 =

1. Define essential single point of f(z)

Soln. A point z =a is an essential singularity if

* + 1. z =a is singular point
		2. z =a should not be a pole (or) removable singularity
1. State Cauchy’s residue theorem

if f(z) is analytic at all point inside and on a single closed curve C except at a finite number of poise z1 , z2 , … ,zn within C then

 sum of the residue of f (z) at z1,z2, ... ,zn

1. Find the zeroes of 

Soln:

 The zero are given by = 0

 

 

 

 The cube roots of unity are the roots.

1. Evaluate ,where C is 1z1=1

Soln:

The poles is z =  lines inside the circle 1z1=1



Where R = residue at z = s

=





1. The Laurent’s Series expansion of  valid in |z| > 1 is

Soln.  

 = 

1. If C is the Circle x2 + y2 = 4 the value of is

Soln.  where c is circle x2+y2=4

 The pole is z = 0 is of order 2 and it is inside the circle |z| = 2

  where R is Reside of z=0zz

 R =

 =

 R= -1

 

 

1. If C is a circle with Z=4+i and radius 4, Then  say true or false?

Soln :

 True, f(z)= is analytic and C is a circle which is closed.

 By cauchy’s integral theorem,the given integral is zero.

1. Expand cos z is a jaylor’s series at Z=

Soln.  

  

  

  

 

 



1. State Cauchy’s Integral theorem

Soln. If  is an analytic and  is continuous inside and on a simple closed curve C, then 

1. Determine the poles of f(z)= and investigate their order

Soln. Z=1 🡪 Pole of Order 1

 Z=2 🡪 Pole of Order 2

1. Evaluate  where C is Circle |z|=2

Soln. The pole is at z=-1 lies inside |z|=2

 where r1=residue at z=-1

 Residue at 

 =e



1. Expand the function  about 

Soln.  

  

  

  

 

 = 

 =

1. Derive residue of f(z), where inside|z|=1

Soin :

 inside the circle |z|=1

 The pole inside the unit circle

 

1. If a=5, then 

Soln. 

 ( is analytic and z = 5 lies outside |z|=2)

1. Obtain the Poles 

Soln. 

 The poles are given by z2+2z+5 =0

 

 z = -1+2i

1. The value of  is

Soln. Let us take c as unit circle z=0 is a double pole

  where r = residue at z=0

 

  = -1

 

1. The Value of  where c is |z| = 1 is

Soln.  the pole is z =0 (double) inside |z| =1

  where r is residue at z = 0

 

 

= -1



1. The value of  when is analytic inside the close curve

[Cauchy’s Integral Theorem]

1. Expand  at z = 1 as a Taylor series

Soln. 

 = -[1-(z-1)]-1

 

1. Coefficient of in laurents expansion of  valid in |z-1| < 1 is equal to

Soln. Put z-1 =u, |u| < 1

 

 

 

 Co efficient of is -1

1. Evaluate where c is |z| = 1.5

Soln. The pole is at z = 1, It is a simple pole and inside c

 

 Where r = residue at z = 1

 

 

 = -1

 

 

1. Express  as a contour integral around the circle |z| = 1

Soln. Put  , 

 , 

 

1. The Laurents expansion of vaid in |z-1|<1 is

Soln. Let u = z-1

 

 

 

1. For a simple pole at z = a the residue of f(z) at z = a where is \_\_\_

Soln. The residue at z = a is 

1. The residue at the pole of the function  are equal

Soln. Since are the poles of order 1

 

 

1. If where c is ellipse x2+y2=4, then the value of is \_\_\_

Soln. 

 The pole is at z=1 which is inside x2+4y2=4

  where r is the residue at z = 1

 Res 

 

 

Part b.

1. Evaluate c∫ 1/(z2+4) dz where c is |z|=3.
2. Obtain Taylor’s series to represent the function z2-1/(z+2)(z+3) in the region |z|<2.
3. Expand 1/ (z-1)(z-2) as a Laurent’s series valid in the regions 1<|z|<2 and |z|>2.
4. Find the residue o Z3/ (z-1)4(z-2)(z-3).
5. If c is the circle |z|=3, evaluate c∫tanz dz.
6. Evaluate c∫dz/ zsinz where c is |z|=1 using cauchy’s residue theorem.
7. Using contour integration evaluate 0∫2π dθ/(13+5sinθ).
8. Using contour integration evaluate 0∫2π cos2θ dθ/(5-4cosθ).
9. Using contour integration evaluate 0∫π dθ/(a2+sin2θ). a>0.
10. Using contour integration evaluate -∞∫∞ xdx/(x+1)(x2+1).
11. Using contour integration evaluate -∞∫∞ (x2-x+2) dx /(x4+10x2+9).
12. Prove that 0∫∞ log(1+x2) / (1+x2) dx = π log2

**UNIT – V LAPLACE TRANSFORMATION**

1. Find 

Soln. 

 

 Taking LCM & on simplification

 

1. Find 

Soln. Formula : 

 

 = Note : 

1. 

Soln. Formula: 

 

 

1. 

Soln. Formula: 

 

 

 Note:

 

1. 

Soln. Formula: 

 

  (cos α, sin α are constants)

 

Note:



1. 

Soln. We know that 

 

1. Find 

Soln. 

 

1. 

Soln.  

 

1. 

Soln:

 =

 , 

1. Soln:

We know that 





 =

 .

1. 

Solution :

 

 f(t)=

 F(s)=L[f(t)]=

 F(s)=

 .

1. Find the Laplace transformation of 

 Solution:

 

 ==

 ==

 ==

 

1. INVERSE LAPLACE TRANSFORMFind 

Soln:

 

1. Find 

Soln :

 

1. Find = 
2. Find 

Soln : 

 = -

 =

 =

1. Find 

Soln :

 

 

1. Find 

Soln. 

 

1. Find 

Soln. 

 

Soln. 

 

 

1. 

Soln. 

 

 

1. 

Soln. 

 

 Sub s = -3

 Sub s = -1

 Subt.

 

 Taking L-1 on both sides

 

1. 

Soln. by 1st Shifting theorem

 

1. Evaluate 

Soln. 

 Hence the given integral is the value of L[t sint] when s = 2

 

 Sub, s = 2

 

1. Evaluate 

Soln. 

 , When s = 0

 

 

  Sub s = 0

 

1. Intial value theorem

Soln. If , then 

 Proof:

 We know that

 

 Taking limit on both side 

 

 

 (i.e)., 

1. Find value theorem

If  

Proof:We know that



Taking limit on both sides 



 



**PART B**

* + 1. Write the change of scale property for Laplace transform with proof and also write inverse Laplace transform for the same.
		2. Write the first shifting property for Laplace transform with proof and also write inverse Laplace transform for the same.
		3. Write the second shifting property for Laplace transform with proof and also write inverse Laplace transform for the same.
		4. State and the convolution theorem.
		5. Find the inverse Laplace transform of (s2 + s – 2)/s(s + 3)(s – 2).
		6. Find the inverse Laplace transform of e-2s/s2(s2 + 1).
		7. Find the inverse Laplace transform of s/(s2 + 1)(s2 + 4)(s2 +9).
		8. Find the inverse Laplace transform of (14s + 10)/(49s2 + 28s + 13).
		9. Find the inverse Laplace transform of (2s3 + 4s2 – s + 1)/(s2)(s2 – s + 2).
		10. Find the inverse Laplace transform of 1/(s4 + 4).
		11. Solve y” + y’ -2 y = 3 cost -11 sin3t ,y(0)=0 and y’(0)=6
		12. Solve using Laplace transforms x’ – y = sint; y’ – x = -cos t given that x =2 , y=0 for t=0.