Inventory Model

1. Alpha industry needs 5400 units per year of a bought out component which will be used in its main product. The ordering cost is Rs. 250 per order and the carrying cost per unit per year is Rs. 30. Find the economic order quantity and the number of orders per year.

Solution:

Demand $D = 5400\, units/year$

Ordering cost $C_0 = Rs. 250/order$

Carrying cost $C_c = Rs. 30\; per\, unit/year$

Economic order quantity

$$EOQ = Q^* = \sqrt{\frac{2C_0D}{C_C}} = \sqrt{\frac{2(250)(5400)}{(30)}} = \sqrt{90000} = 300$$

The number of orders per year

$$= \frac{D}{Q^*} = \frac{5400}{300} = 18\, orders/year$$

2. The demand for an item in a company is 18000 units per year. The company can produce the items at a rate of 3000 units per month. The cost of one set-up is Rs. 500 and the holding cost of 1 unit per month is Rs. 0.15. The shortage cost of one unit is Rs. 20 per year. Determine the optimum manufacturing quantity and the number of shortages. Also determine the manufacturing time and the time between set-ups.

Solution:

Demand $r = 18000\, units/year$

Production rate $k = 3000\, units/month = 3000 \times 12 = 36000\, units/year$

Cost per set-up $C_0 = Rs. 500$

Holding cost $C_c = Rs. 0.15\, per\, unit/month = 0.15 \times 12 = 1.8\, per\, unit/year$

Shortage cost $C_s = Rs. 20\, per\, unit/year$

The optimum manufacturing quantity

$$EBQ = Q^* = \sqrt{\frac{2C_0}{C_C}\frac{kr}{k-r}\frac{C_c + C_s}{C_s}} = \sqrt{\frac{2(500)}{(1.8)}\frac{(36000)(18000)}{(36000-18000)}\frac{(1.8+20)}{(20)}}$$

$$= \sqrt{\frac{2(500)}{(1.8)}\frac{(36000)(18000)}{(18000)}\frac{(21.8)}{(20)}} = \sqrt{21800000} \cong 4669$$

$$Q_1^* = \sqrt{\frac{2C_0}{C_c}\frac{r(k-r)}{k}\frac{C_s}{C_c + C_s}} = \sqrt{\frac{2(500)}{(1.8)}\frac{(18000)(36000-18000)}{(36000)}\frac{(20)}{(1.8+20)}}$$

$$= \sqrt{\frac{2(500)}{(1.8)}\frac{(18000)(18000)}{(36000)}\frac{(20)}{(21.8)}} = \sqrt{4587155.963} \cong 2142$$

$$Q_2^* = \sqrt{\frac{2C_0}{C_s}\frac{r(k-r)}{k}\frac{C_c}{C_c + C_s}} = \sqrt{\frac{2(500)}{(20)}\frac{(18000)(36000-18000)}{(36000)}\frac{(1.8)}{(1.8+20)}}$$

$$= \sqrt{\frac{2(500)}{(20)} \frac{(18000)(18000)}{(36000)} \frac{(1.8)}{(21.8)}} = \sqrt{37155.9633} \cong 193$$

The number of shortages

$$= \frac{r}{Q_2^*} = \frac{18000}{193} \cong 93\, units$$

Manufacturing time

$$= \frac{Q^*}{k} = \frac{4669}{36000} = 0.1297\, year$$

The time between set-ups

$$t^* = \frac{Q^*}{r} = \frac{4669}{18000} = 0.2594\, year$$

3. The annual demand for a product is 100000 units. The rate of production is 200000 units per year. The set-up cost per production run is Rs. 5000 and the variable production cost of each item is Rs. 10. The annual holding cost per unit is 20% of its value. Find the optimum production lot size and the length of the production run.

Solution:

Demand $r = 100000\, units/year$

Production rate $k = 200000\, units/year$

Cost per set-up $C_0 = Rs. 5000$

Production cost $P = Rs. 10/unit$

Holding cost $C_c = 20\%\, of\, P = 0.2 \times 10 = 2\, per\, unit/year$

Optimum production lot size

$$EBQ = Q^* = \sqrt{\frac{2C_0}{C_C} \frac{kr}{k-r}} = \sqrt{\frac{2(5000)}{(2)} \frac{(200000)(100000)}{(200000 - 100000)}}$$

$$= \sqrt{1000000000} \cong 31623\, units$$

Length of the production run

$$t^* = \frac{Q^*}{r} = \frac{31623}{100000} = 0.3162\, year$$

Decision under Uncertainty

1. A retail store desires to determine the optimal daily order size for a perishable item. The store buys the perishable item at the rate of Rs. 80 per kg. and sells at the rate of Rs. 100 per kg. If the order size in more than the demand, the excess quantity can be sold at Rs. 70 per kg in a secondary market; otherwise, the opportunity cost for the store is Rs. 15 experience, it is found that the demand varies from 50 kg to 250 kg in steps of 50 kg. The possible values of the order size from 75 kg to 300 kg in steps of 75 kg. Determine the optimal order size which will maximize the daily profit of the store.

Solution:

Purchase price of the perishable item $= Rs. 80/kg$

Selling price of the perishable item in the primary market $= Rs. 100/kg$
Selling price of the perishable item in the secondary market $= Rs. 70/kg$
Profit in the primary market $P_1 = 100 - 80 = Rs. 20/kg$
Profit in the secondary market $P_2 = 70 - 80 = Rs. -10/kg$
Opportunity cost of not meeting the demand $OC = Rs. 15/kg$

$$Net\ Profit = \begin{cases} D_j \times P_1 + (Q_i - D_j) \times P_2, & if\ Q_i \geq D_j \\ Q_i \times P_1 - (D_j - Q_i) \times OC, & if\ D_j > Q_i \end{cases}$$

$$Net\ Profit = \begin{cases} D_j \times 20 - (Q_i - D_j) \times 10, & if\ Q_i \geq D_j \\ Q_i \times 20 - (D_j - Q_i) \times 15, & if\ D_j > Q_i \end{cases}$$

Where Q_i is the ith order size and D_j is the demand of Jth future state. The corresponding outcomes daily net profits are summarized below.

		Demand D_j				
		50	100	150	200	250
Order size Q_i	75	$50 \times 20 - (75 - 50) \times 10 = 750$	$75 \times 20 - (100 - 75) \times 15 = 1125$	$75 \times 20 - (150 - 75) \times 15 = 375$	$75 \times 20 - (200 - 75) \times 15 = -375$	$75 \times 20 - (250 - 75) \times 15 = -1125$
	150	$50 \times 20 - (150 - 50) \times 10 = 0$	$100 \times 20 - (150 - 100) \times 10 = 1500$	$150 \times 20 - (150 - 150) \times 10 = 3000$	$150 \times 20 - (200 - 150) \times 15 = 2250$	$150 \times 20 - (250 - 150) \times 15 = 1500$
	225	$50 \times 20 - (225 - 50) \times 10 = -750$	$100 \times 20 - (225 - 100) \times 10 = 750$	$150 \times 20 - (225 - 150) \times 10 = 2250$	$200 \times 20 - (225 - 200) \times 10 = 3750$	$225 \times 20 - (250 - 225) \times 15 = 4125$
	300	$50 \times 20 - (300 - 50) \times 10 = -1500$	$100 \times 20 - (300 - 100) \times 10 = 0$	$150 \times 20 - (300 - 150) \times 10 = 1500$	$200 \times 20 - (300 - 200) \times 10 = 3000$	$250 \times 20 - (300 - 250) \times 10 = 4500$

The expected daily net profit with respect to each order size is computed as below:

$$E(Q_1) = \frac{1}{5}(750 + 1125 + 375 - 375 - 1125) = 150$$

$$E(Q_2) = \frac{1}{5}(0 + 1500 + 3000 + 2250 + 1500) = 1650$$

$$E(Q_3) = \frac{1}{5}(-750 + 750 + 2250 + 3750 + 4125) = 2025$$

$$E(Q_4) = \frac{1}{5}(-1500 + 0 + 1500 + 3000 + 4500) = 1500$$

By Laplacian criterion, the expected daily net profit is maximum when the order size is 225 kg. So the store should place an order of 225 kg of the perishable item, daily.

2. The president of a large oil company must decide how to invest the company's Rs. 10 million of excess profits. He could invest the entire sum in solar energy research or he could use the money to research better ways of processing coal so that it will burn more cleanly. His only other option is to put half of this R and D money into solar research and half into coal research. The president estimates 1000 percent return on investment if the solar research is successful and a 500 percent return if the coal research is successful.

(i) Construct a payoff table for the president's R and D investment problem.
(ii) Based on the maximin criterion, what decision should president make?
(iii) Based on the minimax regret criterion, what decision should the president make?

s_1 = neither coal nor solar research is successful
s_2 = Solar research is successful and coal research is not.
s_3 = Coal research is successful and solar research is not

s_4 = both coal and solar research are successful.
$d_!$ = invest in solar R and D only
d_2 = invest in coal R and D only
d_3 = invest 50% in coal and 50% in solar R and D
Solution:
The payoff table for the president's R and D investment problem is given below

	d_1	d_2	d_3
s_1	- 10	-10	-10
s_2	$10 \times \frac{1000}{100} = 100$	- 10	$10 \times \frac{50}{100} \times \frac{1000}{100} - 10 \times \frac{50}{100} = 45$
s_3	- 10	$10 \times \frac{500}{100} = 50$	$-10 \times \frac{50}{100} + 10 \times \frac{50}{100} \times \frac{500}{100} = 20$
s_4	$10 \times \frac{1000}{100} = 100$	$10 \times \frac{500}{100} = 50$	$10 \times \frac{50}{100} \times \frac{1000}{100} + 10 \times \frac{50}{100} \times \frac{500}{100} = 75$

Maximin criterion:

	d_1	d_2	d_3
s_1	- 10	-10	-10
s_2	100	- 10	45
s_3	- 10	50	20
s_4	100	50	75
Min	- 10	- 10	-10

$\text{Maximin} = -10 \text{ Million}$
∴ The president can invest in any one of the following research solar R and D only or coal R and D only or 50% in coal and 50% in solar R and D.

Minimax regret criterion:

	d_1	d_2	d_3
s_1	0	0	0
s_2	0	110	55
s_3	60	0	30
s_4	0	50	25
Max	60	110	55

$$\text{Minimax regret} = 55 \text{ Million}$$

∴ The president can invest half of this R and D money into solar research and half into coal research.

Decision under Risk

1. A manager has a choice between

(i) A risky contract promising Rs. 7 lakhs with probability 0.6 and Rs. 4 lakhs with probability 0.4 and

(ii) A diversified portfolio consisting of two contracts with independent outcomes each promising Rs. 3.5 lakhs with probability 0.6 and Rs. 2 lakhs with probability 0.4. Using the EMV criteria suggest a contract.

Solution:

Risky contract		
Revenue (R_{ij})	Probability(P_{ij})	$P_{ij}\,R_{ij}$
700000	0.6	420000
400000	0.4	160000
Total	1	580000

Diversified portfolio		
Revenue (R_{ij})	Probability(P_{ij})	$P_{ij}\,R_{ij}$
350000	0.6	210000
200000	0.4	80000
Total	1	290000

The expected Revenue

$$ER_i = \frac{\sum_j P_j R_j}{\sum_j P_j}, \quad ER_1 = \frac{580000}{1} = 580000, \quad ER_2 = \frac{290000}{1} = 290000$$

Here the expected revenue of a risky contract is more than the expected revenue of diversified portfolio. Therefore the manager suggests a risky contract.

Decision tree

1. Amar company is currently working with a process which after paying for materials, labour etc., brings a profit of Rs. 12,000. The following alternatives are made available to the company:

(i) The company can conduct research (R_1) which is expected to cost Rs. 10,000 having 90% chances of success. If it proves a success the company gets a gross income of Rs. 25,000.

(ii) The company can conduct research (R_2) which is expected to cost Rs. 8,000 having a probability of 60% success, the gross income will be Rs. 25,000.

(iii) The company continues the current process.

Because of limited resources, it is assumed that only one of the two types of research can be carried out a time. Use decision tree analysis to locate the optimal strategy for the company.

Solution:

EMV of decision node 4 $= \text{Max.Rs.}[12000] = \text{Rs.}\,12000$

EMV of decision node 5 $= \text{Max.Rs.}[12000] = \text{Rs.}\,12000$

EMV of chance node C= $\text{Rs.}[0.4 \times 12000 + 0.6 \times 25000] = \text{Rs.}\,19800$

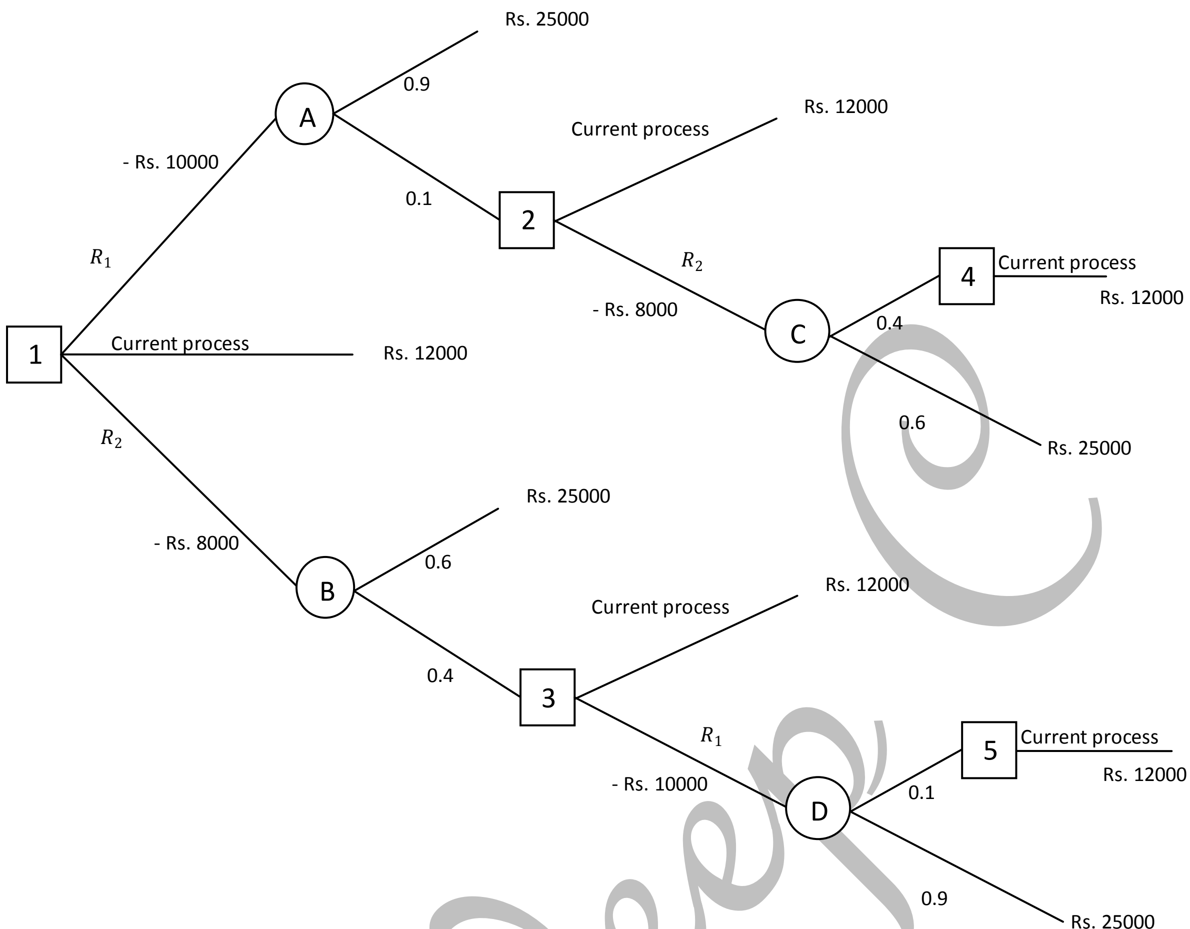
EMV of chance node D= $\text{Rs.}[0.1 \times 12000 + 0.9 \times 25000] = \text{Rs.}\,23700$

EMV of decision node 2 $= \text{Max.Rs.}[12000, (19800 - 8000)] = \text{Rs.}\,12000$

EMV of decision node 3 $= \text{Max.Rs.}[12000, (23700 - 10000)] = \text{Rs.}\,13700$

EMV of chance node A $= \text{Rs.}[0.9 \times 25000 + 0.1 \times 12000] = \text{Rs.}\,23700$

EMV of chance node B $= \text{Rs.}[0.6 \times 25000 + 0.4 \times 13700] = \text{Rs.}\,20480$



EMV of decision node 1 $= \text{Max. Rs.}\,[\,(23700 - 10000), 12000, (20480 - 8000)] = \text{Rs.}\,137000$
Thus the company can conduct research R_1 to earn maximum expected profit of Rs. 13,700.

Simulation

1. A baker has to supply 200 pizzas every day to their outlet situated in city bazzar. The productions of pizzas vary due to availability of raw materials and labour for which probability distribution of production by observation made is as follows:

Production per day	196	197	198	199	200	201	202	203	204
Probability	0.06	0.09	0.10	0.16	0.20	0.21	0.08	0.07	0.03

Simulate and find the average number of pizzas produced more than the requirement and average number of shortage of pizzas supplied to the outlet. Use the random number 97, 02, 80, 66, 96, 55, 50, 29, and 58 for this problem.
Solution:

Production per day	Probability	Cumulative Probability	Random number interval
196	0.06	0.06	0 – 5
197	0.09	0.15	6 – 14
198	0.10	0.25	15 – 24
199	0.16	0.41	25 – 40
200	0.20	0.61	41 – 60
201	0.21	0.82	61 – 81
202	0.08	0.90	82 – 89
203	0.07	0.97	90 – 96
204	0.03	1	97 – 99

Day	Random Number	Production per day	Pizzas produced more than the requirement	Shortage of pizzas supplied to the outlet
1	97	204	4	-
2	2	196	-	4
3	80	201	1	-
4	66	201	1	-
5	96	203	3	-
6	55	200	-	-
7	50	200	-	-
8	29	199	-	1
9	58	200	-	-
		Total	9	5

$$The\ average\ number\ of\ pizzas\ produced\ more\ than\ the\ requirement = \frac{9}{9} = 1\ per\ day$$

$$The\ \ average\ number\ of\ Shortage\ of\ pizzas\ supplied\ to\ the\ outlet = 5/9 = 0.56\ per\ day$$

2. In a plant expected to manufacture 100 cars per day, deviations occur due to a variety of reasons. The probabilities associated with the number of production units per day have been determined using past data

Production per day	95	96	97	98	99	100	101	102	103	104	105
Probability	0.03	0.05	0.07	0.1	0.15	0.2	0.15	0.1	0.07	0.05	0.03

The ship carrying the cars can accommodate up to 101 cars per day. Using the random 97, 2, 80, 66, 96, 55, 50, 29, 58, 51, 4, 86, 24, 39, 47 simulate the production run for a period of 15 day. Also determine the average number of cars waiting to be shipped and average number of empty spaces in the ship.

Solution:

Production per day	Probability	Cumulative Probability	Random number interval
95	0.03	0.03	0 – 2
96	0.05	0.08	3 – 7
97	0.07	0.15	8 – 14
98	0.1	0.25	15 – 24
99	0.15	0.40	25 – 39
100	0.2	0.60	40 – 59
101	0.15	0.75	60 – 74
102	0.1	0.85	75 – 84
103	0.07	0.92	85 – 91
104	0.05	0.97	92 – 96
105	0.03	1	97 – 99

Day	Random Number	Production per day	Cars waiting to be shipped	Empty spaces in the ship
1	97	105	4	-
2	2	95	-	2
3	80	102	1	-
4	66	101	1	-
5	96	104	4	-
6	55	100	3	-
7	50	100	2	-
8	29	99	-	-
9	58	100	-	1
10	51	100	-	1
11	4	96	-	5
12	86	103	2	-
13	24	98	-	1
14	39	99	-	2
15	47	100	-	1
		Total	17	13

$$\text{The average number of cars waiting to be shipped} = \frac{17}{15} = 1.13\ per\ day$$

$$\text{The average number of empty spaces in the ship} = \frac{13}{15} = 0.87\ per\ day$$

3. A dentist schedules all his patients for 30 minute appointments. The following summary shows the various categories of work, their probability and time actually needed to complete the work.

Category of service	Filling	Crown	Cleaning	Extract	Checkup
Time required(Minutes)	45	60	15	45	15
Probability	0.4	0.15	0.15	0.1	0.2

Simulate the dentist's clinic for four hours and determine the average waiting time for the patients as well as the idleness of the doctor. Assume that all the patients show up at the clinic at exactly their scheduled arrival times, starting at 8 A.M. Use the following random numbers 40, 82, 11, 34, 25, 66, 17 and 79.

Solution:

Category	Time (Minutes)	Probability	Cumulative Probability	Random number interval
Filling	45	0.40	0.40	0 – 39
Crown	60	0.15	0.55	40 – 54
Cleaning	15	0.15	0.70	55 – 69
Extraction	45	0.10	0.80	70 – 79
Checkup	15	0.20	1	80 – 99

Patient no.	Random no.	Arrival time	Dentist's treatment		Waiting time on the part of patient(Minutes)	Idle time for the dentist
			starts	Ends		
1	40	8.00	8.00	9.00	-	-
2	82	8.30	9.00	9.15	30	-
3	11	9.00	9.15	10.00	15	-
4	34	9.30	10.00	10.45	30	-
5	25	10.00	10.45	11.30	45	-
6	66	10.30	11.30	11.45	60	-
7	17	11.00	11.45	12.30	45	-
8	79	11.30	12.30	1.15	60	-
				Total	285	0

$$\text{Average waiting time for the patients} = \frac{285}{8} = 35.625 \text{ minutes}$$

$$\text{Average idleness of the dentist} = \text{Nil}$$