SRI RAMAKRISHNA INSTITUTE OF TECHNOLOGY, COIMBATORE- 10

DEPARTMENT OF SCIENCE AND HUMANITIES

B.E - EEE & CIVIL

SUBJECT: NUMERICAL METHODS (SEMESTER - VI)

UNIT - IV

INITIAL VALUE PROBLES FOR ORDINARY DIFFERENTIAL EQUATIONS

01. Write down the fourth order Taylor's algorithim.

Answer :

Let $y' = f(x, y)\ \ with\ \ y(x_0) = y_0$

Then the Taylor algorithim is given by

$$y(x_1) = y_0 + \frac{h}{1!}\, y_0' + \frac{h^2}{2!}\, y_0'' + \frac{h^3}{3!}\, y_0''' + \frac{h^4}{4!}\, y_0^{iv} + \cdots$$

$$where\quad x_1 = x_0 + h \quad and\quad y_0^{(r)} = \frac{d^r y}{dx^r}\quad at\quad (x_0, y_0).$$

02. What are the merits and demerits of the Taylor series method of solution?

Answer :

It is a powerful single step method.

It is the best method if the expression for higher order derivtives are simpler.

The major demerit of this method is the evaluation of higher order derivatives become tedious for complicated algebric expressions.

03. Given $y' = x + y,\ y(0) = 1.$ Find $y(0) = 1$ By Taylor's method.

Answer :

$$y' = x + y\ ;\ \ y'' = 1 + y';\ \ y''' = y'' \dots\dots$$

$$x_0 = 0, \qquad y_0 = 0.\ Then\ \ y(0.1) = y_1 = y_0 + \frac{h}{1!}\, y_0' + \frac{h^2}{2!}\, y_0'' + \cdots$$

$$when\ \ h = 0.1. \qquad \therefore\ \ y(0.1) = 1 + 0.1 + \frac{0.01}{2}\,(2) + \frac{0.001}{6} 6\,(2)$$

$$\Rightarrow\ \ y(0.1) = 1.1103$$

04. Find $y(0.1)\ by$ Euler's method, given that $y' = 1 - y,\ y(0) = 0.$

Answer :

$$y_1 = y_0 + h\, f(x_0, y_0) = 0.01[1 - 0] = 0.1 \quad \Rightarrow\ \ y(0.1) = 0.1$$

05. Using Euler's method compute for $x = 0.1\ \ \&\ \ 0.2\ \ with\ \ h = 0.1\ \ \ given$

$y' = y - \frac{2x}{y}$, $y(0) = 1.$

Answer :

$$y_1 = 1 + 0.1[1 - 0] = 1.10 \quad \Rightarrow \quad y(0.1) = 1.10$$

$$y_2 = 1.1 + 0.1\left[1.1 - \frac{0.2}{1.1}\right] = 1.19 \quad \Rightarrow \quad y(0.2) = 1.19$$

06. Find $y(0.1)$ *by* Euler's method, given that $y' = x + y,\ y(0) = 1.$

Answer :

$$y_1 = 1 + 0.1[0 + 1] = 1.10 \quad \Rightarrow \quad y(0.1) = 1.10$$

07. Given $y' + y = 0$ *and* $y(0) = 1.$ Find $y(0.01)$ *and* $y(0.02)$ by Euler's method

Answer :

$$y_1 = y_0 + h\,f(x_0, y_0) = 0.01[1 - 0] = 0.1 \quad \Rightarrow \quad y(0.1) = 0.1$$

08. Using Euler's method compute for $x = 0.1\ \&\ 0.2\ with\ h = 0.1\ given$

$y' = y - \frac{2x}{y}$, $y(0) = 1.$

Answer :

$$y_1 = 1 + 0.01[-1] = 0.09 \quad \Rightarrow \quad y(0.01) = 0.09$$

$$y_1 = 0.99 + 0.01[-0.99] = 0.9801 \quad \Rightarrow \quad y(0.02) = 0.9801$$

09. State the algorithim for modified Euler's method, to solve $\frac{dy}{dx} = f(x, y),\ y(x_0) = y_0.$

Answer :

$$y_{n+1}^{(1)} = y_n + h\,f(x_n, y_n)$$

$$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f\left(x_{n+1}, y_{n+1}^{(1)}\right)\right]$$

$$where \quad n = 0,1,2, \dots \quad and \quad x_{n+1} = x_n + h$$

10. State Rung-Kutta fourth order formulae for solving $\frac{dy}{dx} = f(x, y),\ y(x_0) = y_0.$

Answer :

$$y_1 = y(x_0 + h) = y_0 + \frac{1}{6}\,[k_1 + 2k_2 + 2k_3 + k_4] \quad where$$

$$k_1 = h\,f(x_0, y_0)$$

$$k_2 = h\,f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$k_3 = h\,f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$

$$k_4 = h\,f(x_0 + h, y_0 + k_3)$$

11. What are th distinguish property for Rung-Kutta methods?

Answer :

(1). These methods do not require the higher order derivatives and requires only the function values at different points.

(2). To evaluate y_{n+1}, we need only y_n but not previous of y's.

(3). The solution by these methods agree with Taylor series solution upto the terms of h^r Where r is the order of Runge-Kutta method.

12. Which is the better Taylor series method or Runge – Kutta method? Why?

Answer :

Runge-Kutta method is better since higher order derivatives of y are not required. Taylor's series method involves use of higher oder derivatives which may be difficult in case of complicated algebric functions.

13. State the order of error in R-K method of fourth order.

Answer :

Error of fourth order method is $O(h^2)$ where h is the interval of differencing.

14. State Milne's Predictor and Corrector formula.

Answer :

Predictor : $y_{n+1}^{p} = y_{n-3} + \frac{4h}{3}\left[2\, y'_{n-2} - y'_{n-1} + 2\, y'_n\right]$

Corrector : $y_{n+1}^{c} = y_{n-1} + \frac{h}{3}\left[\, y'_{n-2} + 4y'_{n-1} + \; y'_{n+1}\right]$

15. State Adam's Predictor and Corrector formula.

Answer :

Predictor : $y_{n+1,P} = y_n + \frac{h}{24}\left[55\, y'_n - 59y'_{n-1} + 37\, y'_{n-2} - 9y'_{n-3}\right]$

Corrector : $y_{n+1,C} = y_n + \frac{h}{24}\left[9\, y'_{n+1} + 19y'_n - 5\, y'_{n-1} + y'_{n-2}\right]$

16. Write the predictor error and corrector error in Milne's method.

Answer :

Predictor error $= \frac{14}{45}\, h^5 f^{iv}(\mathcal{E})$

Corrector error $= -\frac{h^5}{90}\, y^{iv}(\mathcal{E})$

17. Distinguish Single – step and Multi – step methods.

Answer :

Single – step methods : To find y_{n+1} , the information at y_n is enough.

Multi – step methods : To find y_{n+1} , the past four values $y_{n-3}, y_{n-2}, y_{n-1}$ *and* y_n are needed.

18. State the finite approximations for y' & y'' with error terms.

Answer :

$$y_i = y(x_i) \quad and \quad x_{i+1} = x_i + h \,, \qquad i = 0,1,2,\dots n$$

$$Then \qquad y_i' = \frac{y_{i+1} - y_{i-1}}{2h}, \qquad Error\ = O(h^2)$$

$$y_i'' = \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2}, \qquad Error\ = O(h^2)$$

19. Solve $x\, y'' + y = 0,\ y(1) = 1, y(2) = 2\ \ with\ \ h = 0.5.$

Answer :

Finite difference scheme : $4\,x_i(y_{i+1} - 2\,y_i + y_{i-1}) + y_i = 0$

$$For\ \ i = 1, \qquad 4\,x_1(y_2 - 2\,y_1 + y_0) + y_1 = 0$$

$$\Rightarrow 11\,y_1 = 18 \qquad \Rightarrow\ y_1 = \frac{18}{11} = 1.6364$$

$$\Rightarrow y(1.5) = 1.6364$$

20. Give the finite difference scheme to solve $u_{xx} + u_{yy} = 0$ numerically.

Answer :

For square mesh of sides

$$\Delta x = h = \Delta y,$$

$$u_{i,j} = \frac{1}{4}\left[u_{i-1,j} + u_{i+1,j-1} + u_{i,j+1}\right] \quad where \quad u_{i,j} = u(x_i, y_i)$$

21. What is the purpose of Leibmann's process?

Answer :

The purpose of Leibmann's process is to find the solution of Laplace equation $u_{xx} + u_{yy} = 0$

By iteration over a square with boundary values.

22. Express $u_{xx} + u_{yy} = f(x,y)$ in finite difference scheme.

Answer :

For a square mesh with interval of differencing h , we have

$$u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1} - 4u_{i,j} = h^2\, f(ih, jh)$$

Part – B

1. Using Taylor series method find y at $x = 0.1$ if $\frac{dy}{dx} = x^2y - 1,\ y(0) = 1.$

2. Solve $y' = x + y;\ y(0) = 1$ by Taylor's series method. Find the vlues $y\ at\ x = 0.1\ and\ x = 0.2.$

3. Solve $\frac{dy}{dx} = y^2 + x^2$ with $y(0) = 1.$ Use Taylor's series at $x = 0.2\ and\ 0.4.$ Find $x = 0.1$

4. Using Taylor series method find $y\ at\ x = 0.1$ correct to four decimal places from $\frac{dy}{dx} = x^2 - y\ ;\ y(0) = 1,\ with\ h = 0.1.$ Compute terms up to x^4.

5. Using Taylor series method, compute $y(0.2)$ & $y(0.4)$ correct to four decimal places given $\frac{dy}{dx} = 1 - 2\,x\,y\ \ and\ y(0) = 0.$

6. Using Euler's method find $y(0.2)$ & $y(0.4)$ from $\frac{dy}{dx} = x + y,\ y(0) = 1\ \ with\ h = 0.2.$

7. Using Euler's method solve $y' = x + y + x\,y,\ y(0) = 1$ compute $y\ at\ x = 0.1,$ by taking $h = 0.05.$

8. Using Euler's method find $y(0.3)$ of $y(x)$ satisfies the initial value problem.
$\frac{dy}{dx} = \frac{1}{2}(x^2 + 1)y^2,\quad y(0.2) = 1.1114.$

9. Solve $y' = 1 - y,\ y(0) = 0$ by modified Euler's method.

10. Using modified Euler's method, find $y(0.1)$ if $\frac{dy}{dx} = x^2 + y^2,\ y(0) = 1$.

11. Given $\frac{dy}{dx} = x^3 + y,\ y(0) = 2.$ Compute $y(0.2), y(0.4)$ & $y(0.6)$ by Runge-Kutta method of fourth order.

12. Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1\ \ at\ x = 0.2.$

13. Using Runge-Kutta method of fourth order, find $y(0.1)\ and\ y(0.2)$ for the initial value problem $\frac{dy}{dx} = x + y^2,\ y(0) = 1.$

14. Apply fourth order Runge-Kutta method to determine $y(0.2)$ with $h = 0.1$ from $\frac{dy}{dx} = x^2 + y^2,\ y(0) = 1.$

15. Find $y(0.8)$ given that $y' = y - x^2,\ y(0.6) = 1.7379$ by using Runge-Kutta method of fourth order. Take $h = 0.1$

16. Determine the value of $y(0.4)$ using Milne's method given $y' = x\,y + y^2,\ y(0) = 1;$ use Taylor series method to get the values of $y(0.1),\ y(0.2)\ and\ y(0.3).$

17. Using Milne's method find $y(0.2)$ if $y(x)$ is the solution of $\frac{dy}{dx} = \frac{1}{2}(x + y),\ given$

$y(0) = 2,\ y(0.5) = 2.636,\ y(1) = 3.595\ \ \&\ \ y(1.5) = 4.968$

18. Solve $y' = x - y^2,\ \ 0 \le x \le 1,\ \ y(0) = 0,\ \ y(0.2) = 0.02,\ \ y(0.4) = 0.0795,$ $y(0.6) = 0.1762$ by Milne's method to find $y(0.8)$ & $y(1)$.

19. Using Milne's method find $y(4.4)\ \ given\ \ 5\,x\,y' + y^2 - 2 = 0\ \ given\ \ y(4) = 1,\ y(4.1) = 1.0049,\ \ y(4.2) = 1.0097\ \ \&\ \ y(4.3) = 1.0143$

20. Given $\frac{dy}{dx} = x^3 + y,\ y(0) = 2.$ The values of $y(0.2) = 2.073,\ y(0.4) = 2.452\ \ \&\ \ y(0.6) = 3.023$ are got by R-K method of fourth order. Find $y(0.8)$ by Milne's Predictor corrector method taking $h = 0.2$