SRI RAMAKRISHNA INSTITUTE OF TECHNOLOGY, COIMBATORE- 10
DEPARTMENT OF SCIENCE AND HUMANITIES
COMMON TO BE (EEE & CIVIL)
SUBJECT: NUMERICAL METHODS (SEMESTER – IV)

UNIT II

INTERPOLATION & APPROXIMATION

1. State Lagrange's interpolation formula.

Answer :

Let $y = f(x)$ be a function which takes the values

$y_0, y_1, \dots \dots . y_n$ *corresponding to* $x = x_0, x_1, x_2, \dots \dots \dots$

Then , Lagrange's interpolation formula is

$$y(x) = \frac{(x-x_1)(x-x_2)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)\dots(x_0-x_n)}\ y_0 + \frac{(x-x_0)(x-x_2)\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)\dots(x_1-x_n)}\ y_1$$
$$+ \frac{(x-x_0)(x-x_1)\dots(x-x_n)}{(x_2-x_0)(x_2-x_1)\dots(x_2-x_n)}\ y_2$$
$$+ \cdots \dots . + \frac{(x-x_0)(x-x_1)\dots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\dots(x_n-x_{n-1})}\ y_n$$

2. What is the Lagrange's interpolation formula to find y, if three sets of values $(x_0, y_0), (x_1, y_1)$ & (x_2, y_2) are given.

Answer :

$$y(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}\ y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}\ y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}\ y_2$$

3. What is the assumption we make when Lagrange's formula is used?

Answer :

Lagrange's interpolation formula can be used whether the vales of x, the independent variable are equally spaced or not whether the difference of y become smaller or not.

4. What advantages has Lagrange's interpolation formula over Newton?

Answer :

The forward and backward interpolation formulae of Newton can be used only when the values of the independent variable x are equally spaced can also be used when the differences of the independent variable y become smaller ultimately. But Lagrange's interpolation formula can be used whether the values of x, the independent variable are equally spaced or not and whether the difference of y become smaller or not.

5. What is the disadvantage in practice in applying Lagrange's interpolation formula?

Answer :

Through Lagrange's interpolation formula is simple and easy to remember, its application is not speedy. It requires close attention to sign and there is always a chance of committing some error due to a number of positive and negative signs in the numerator and the denominator.

6. What is inverse interpolation ?

Answer :

Suppose we are given a table of vales of x and y. Direct interpolation is the process of finding the values of y corresponding to a value of x, not present in the table. Inverse interpolation is the process of finding the values of x corresponding to a value of , not present in the table.

7. State Lagrange's inverse interpolation formula.

Answer :

$$x = \frac{(y-y_1)(y-y_2)\ldots(y-y_n)}{(y_0-y_1)(y_0-y_2)\ldots(y_0-y_n)}\ x_0 + \frac{(y-y_0)(y-y_2)\ldots(y-y_n)}{(y_1-y_0)(y_1-y_2)\ldots(y_1-y_n)}\ x_1$$
$$+\ \frac{(y-y_0)(y-y_1)\ldots(y-y_n)}{(y_2-y_0)(y_2-y_1)\ldots(y_2-y_n)}\ x_2$$
$$+\cdots\ldots.+\frac{(y-y_0)(y-y_1)\ldots(y-y_{n-1})}{(y_n-y_0)(y_n-y_1)\ldots(y_n-y_{n-1})}\ x_n$$

8. Define 'Divided Differences'.

Answer :

Let the function $y = f(x)$ take the values $f(x_0), f(x_1), \ldots f(x_n)$ corresponding to the values $x_0, x_1, x_2, \ldots \ldots . x_n$ of the argument x where $x_1 - x_0,\ x_2 - x_1, \ldots \ldots x_n - x_{n-1}\ need$ not necessarily be equal.

The first divided difference of $f(x)$ for the arguments x_0, x_1 is

$$f(x_0, x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

Similarly

$$f(x_1, x_2) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

9. Form the divided for the following data

x :	2	5	10
y :	5	29	109

Solution : The divided difference table is as follows

x :	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$
2	5		
		$\frac{29-5}{5-2} = 8$	
5	29		$\frac{16-8}{10-2} = 1$
		$\frac{109-29}{10-5} = 16$	
10	109		

9. Form the divided for the following data

x :	5	15	22
y :	7	3629	160

Solution : The divided difference table is as follows

x :	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$
5	7		
		$\frac{36-7}{15-5} = 2.9$	
15	36		$\frac{17.7-2.9}{22-5} = 2.114$
		$\frac{160-36}{22-15} = 17.7$	
22	160		

10. Give the Newton's divided difference formula.

Solution :

$$f(x) = f(x_0) + (x - x_0)f(x_0,x_1) + (x - x_0)\,(x - x_1)f(x_0,x_1,x_2)$$
$$+ (x - x_0)\,(x - x_1)\,(x - x_2)f(x_0,x_1,x_2,x_3)$$
$$+ \cdots \ldots \ldots + (x - x_0)(x - x_1) \ldots (x - x_n)\; f(x_0,x_1, \ldots x_{n-1})$$

11. State any properties of divided differences.

Solution :

(1). The divide difference are symmetrical in all their arguments. That is the value of any difference is independent of the order of the arguments.

(2). The divided difference of the sum or difference of two functions is equal to the sum or difference of the corresponding separate divided differences.

12 . What is a cubic spline?

Solution :

A cubic polynomial which has continuous slope and curvature is called a cubic spline.

13. What is a natural cubic spline?

Solution :

A cubic spline fitted to the given data such that the end cubic's approach linearity at their extremities is called a natural cubic spline.

14. State the conditions required for a natural cubic spline?

Solution :

A cubic spline $g(x)$ fits to each points in continuous and is continuous in slope and curvature such that $x_0 = g_0''(x_0) = 0$ *and* $S_n = g_{n-1}''(x_n) = 0$ is called a natural cubic spline. Let us assume that $(x_i, y_i), i = 0,1, \ldots n$ are data points.

15. Derive Newton's Backward difference formula by using operator method.

Solution :

$$P_n(x) = P_n(x_n + vh) = E^v P_n(x_n)$$
$$= (1-\nabla)^{-v} y_n \ \ since \ \ E = (1-\nabla)^{-1}$$
$$= \left[1 + v\,\nabla + \frac{v(v+1)}{2!}\,\nabla^2 + \frac{v(v+1)(v+2)}{3!}\,\nabla^3 + \cdots\right] y_n$$
$$= y_n + v\,\nabla y_n + \frac{v(v+1)}{2!}\,\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\,\nabla^3 y_n + \cdots$$

$where \ \ v = \frac{x - x_n}{h}$

16. Derive Newton's Forward difference formula by using operator method.

(or) State Gregory – Newton Forward difference interpolation formula.

Solution :

$$P_n(x) = P_n(x_0 + uh) = E^u P_n(x_0)$$
$$= (1+\Delta)^u y_0$$

$$= \left[1 + v\,\nabla + \frac{v(v+1)}{2!}\,\nabla^2 + \frac{v(v+1)(v+2)}{3!}\,\nabla^3 + \cdots\right] y_n$$

$$= y_0 + \frac{u}{1!}\,\Delta y_0 + \frac{u(u-1)}{2!}\,\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\,\Delta^3 y_0 + \cdots$$

$where\ u = \frac{x-x_0}{h}$

17. When Newton's Backward interpolation formula used.

Solution :

The formula is used mainly to interpolate the values of y near the end of a set of tabular values and also for extrapolating the values of y a short distance ached (to the right) of y_n.

18. Newton's Backward interpolation formula used only for ....?

Solution :

Equidistant intervals (or) Equal intervals.

19. Say True or False.

Newton's interpolation formulae are not suited to estimate the value of a function near the middle of a table.

Answer : True.

20. Say True or False.

Newton's forward and Newton's Backward interpolation formulae are applicable for interpolation near the beginning and end respectively of tabulated values.

Answer : The statement is True.

21. When will we use Newton's forward interpolation formula ?

Solution :

The formula is used to interpolate the values of y near the beginning of the table value and also for extrapolating the values of y short distance (to the left) ahead of y_0.

Part – B

1. Find the polynomial $f(x)$ by using Lagrange's formula and hence find $f(3)$ for

x:	0	1	2	5
$f(x)$:	2	3	12	147

2. Using Lagrange's interpolation formula to calculate the profit in the year **2000** from the following data.

$Year$:	1997	1999	2001	2002
$Profit$:	43	65	159	248

3. Find the third degree polynomial $f(x)$ satisfying the following data.

x:	1	3	5	7
$f(x)$:	24	120	336	720

4. Using Lagrange's interpolation formula find $y(2)$ from the following data.

x:	0	1	3	4	5
$f(x)$:	0	1	81	256	625

5. Using Lagrange's interpolation formula find $f(4)$ given that

$f(0) = 2,\ f(1) = 3,\ \ f(2) = 12,\ \ f(15) = 3587$

6. Find $f(x)$ as a polynomial in x for the following data by Newton's divided difference formula.

x:	- 4	- 1	0	2	5
$f(x)$:	1245	33	5	9	1335

7. Using Newton's divided difference formula, find $u(3)$ given that

$u(1) = -26,\ \ u(2) = 12,\ \ u(4) = 256,\ \ u(6) = 844.$

8. Find $f(8)$ Newton's divided difference formula from the data :

x:	4	5	7	10	11	13
$f(x)$:	48	100	294	900	1210	2028

9. Using Newton's divided difference formula, find $f(3)$ from the data

x:	0	1	2	4	5
$f(x)$:	1	14	15	5	6

10. Using Newton's divided difference formula, find the missing value from the table

x:	1	2	4	5	6
$f(x)$:	14	15	5	--	9

11. Using Newton's Forward interpolation formula , find the polynomial $f(x)$ satisfying the following data. Hence evaluate $y\ at\ x = 5$.

x:	4	6	8	10
$f(x)$:	1	3	8	10

12. Using Newton's Forward interpolation formula , find the polynomial $f(x)$ satisfying the following data. Hence find $f(2)$.

x:	0	5	10	15
$f(x)$:	14	379	1444	3584

13. A third degree polynomial passes through the points $(0,-1),\ (1,1),\ (2,1)\ \ and\ \ (3,-2)$ using Newton's Forward interpolation formula find the polynomial. Hence find the value at 1.5.

14. Using Newton's Backward difference formula to construct an interpolating polynomial of degree 3 for the data :

$f(-0.75) = -0.071811250,\ f(-0.5) = -0.024750,$

$f(-0.25) = 0.33493750,\ \ f(0) = 1.10100.$ Hence find $f\left(-\frac{1}{3}\right)$.

15. The following data are taken from the steam table :

$Temp.\ \ {}^0C$:	140	150	160	170	180

$Pressure\ kgf/cm^2$	3.685	4.854	6.302	8.076	10.225

16. From the following table find the value of $\tan(0.12)$ and $\tan(0.28)$

x :	0.10	0.15	0.20	0.25	0.30
$y = \tan x$:	0.1003	0.1511	0.2027	0.2533	0.3093

17. Obtain the cubic spline approximation for the function $y = f(x)$ from the following data $y_0'' = y_3' = 0$.

x :	-1	0	1	2
y :	-1	1	3	35

18. Obtain natural cubic spline which agrees with $y(x)$ at the set of data points given below.

x :	2	3	4
y :	11	49	123

19. Fit a natural cubic spline to the following data :

x :	-1	0	1
y :	-4	3	2

20. Fit a natural cubic spline to the following data assuming that $y''(1) = 0$ & $y''(3) = 0$.

x :	1	2	3
y :	-6	-1	16