UNIT-IV

NON-PARAMETRIC METHODS

χ^2 –Test of Goodness of Fit

If $O_i \ (i = 1,2,..n)$ is a set of observed frequencies and $E_i \ (i = 1,2,\dots n)$ is the corresponding se of expected frequencies, then

$$\chi^2 = \sum_{i=1}^{n} \frac{(\boldsymbol{O_i - E_i})^2}{\boldsymbol{E_i}}$$

follows chi-square distribution with $n-1$ degrees of freedom.

Conditions for the validity of χ^2 test

1. The experimental data (samples) must be independent to each other.
2. The total frequency (no. of observations in the sample) must be large, say ≥ 50.
3. All the individual data's should be greater than 5.
4. The no. of classes n must lies in $4 \leq n \leq 16$.

Example: 1

The following table gives the number of air accidents that occur during the various days of a week. Find whether the accidents are uniformly distributed over the week.

Days	Sun	Mon	Tue	Wed	Thu	Fri	Sat
No. of Accidents	14	16	8	12	11	9	14

Solution:

Null Hypothesis $(\boldsymbol{H_0})$: The accidents are uniformly distributed over the week.

Alternative Null Hypothesis $(\boldsymbol{H_1})$: The accidents are not uniformly distributed over the week.

The test statistic is given by

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} \quad \chi^2 \ distribution \ with \ (n-1) \ d.o.f$$

$$E_i = \frac{total \ no. of \ absorvations}{n} = \frac{84}{7} = 12 \qquad N = 84, \ \ n = 7$$

$$E_i = 12$$

Day	Observed freq	Expected freq	$(O_i - E_i)$	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
Sun	14	12	2	4	0.333333
Mon	16	12	4	16	1.333333
Tue	8	12	-4	16	1.333333
Wed	12	12	0	0	0

Thu	11	12	-1	1	0.083333
Fri	9	12	-3	9	0.75
Sat	14	12	2	4	0.333333
	84	84			4.166

$$\chi^2 = \mathbf{4.166}$$

Table value of $\chi^2_{0.05}$ with $n-1 = 7-1 = 6$ d.o.f is 12.59.

Conclusion:

Since $\chi^2 < \chi^2_{0.05}$, we accept null hypothesis. That is the air accidents are uniformly distributed over the week.

Example: 2

The following figures show the distribution of digits in numbers chosen at random from the following directory:

Digits	0	1	2	3	4	5	6	7	8	9	Total
Frequency	1026	1107	997	966	1075	933	1107	972	964	853	10000

Test whether the digits may be taken to occur equally frequently in the directory.

Solution:

Null Hypothesis $(\boldsymbol{H_0})$: The accidents are uniformly distributed over the week.

Alternative Null Hypothesis $(\boldsymbol{H_1})$: The accidents are not uniformly distributed over the week.

The test statistic is given by

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} \quad \chi^2 \ distribution\ with\ (n-1)\ \ d.o.f$$

$$E_i = \frac{total\ no.of\ observations}{n} = \frac{10000}{10} = 1000 \qquad N = 10000, \quad n = 10$$

$$E_i = 1000$$

Day	$Observed\ freq$	$Expected\ freq$	$(O_i - E_i)$	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
0	1026	1000	26	676	0.676
1	1107	1000	107	11449	11.449
2	997	1000	−3	9	0.009
3	966	1000	−34	1156	1.156
4	1075	1000	75	5625	5.625
5	933	1000	−67	4489	4.489
6	1107	1000	107	11449	11.449
7	972	1000	−28	784	0.784
8	964	1000	−36	1296	1.296
9	853	1000	−147	21609	21.609
	10000	10000			58.542

$$\chi^2 = \mathbf{58.542}$$

Table value of $\chi^2_{0.05}$ with $n-1 = 10-1 = 9$ d.o.f is 16.919.

Conclusion:

Since $\chi^2 > \chi^2_{0.05}$, we reject null hypothesis. That is the digits are not uniformly distributed over the directory.

χ^2 –Test of Independence of Attributes

Under the null hypothesis H_0: the attributes A and B are independent and we calculate the expected frequencies E_{ij} for various cells using the following formula.

$$E_{ij} = \frac{R_i C_j}{N} \;\; ; \;\; i = 1,2,\ldots r \;\; and \;\; j = 1,2,\ldots.s$$

To conduct the test, we compute

$$\chi^2 = \sum_{i=1}^{r}\sum_{j=1}^{s}\frac{\left(O_{ij}-E_{ij}\right)^2}{E_{ij}} \qquad \chi^2 \; distribution \, with \;\; n = (r-1)(s-1)-1 \quad d.o.f$$

Remark:

For the 2×2 contingency table with the cell frequencies a, b, c and d, the χ^2 value is given by

$$\chi^2 = \frac{N(ad-bc)^2}{(a+c)(b+d)(a+b)(c+d)}; \;\; N = a+b+c+d;$$

Example: 3

Two researchers adopted different sampling techniques while investigating the same group of students to find the number of students falling in different intelligence levels. The results are as follows.

Researches	Below Average	Average	Above average	Genius	
X	86	60	44	10	200
Y	40	33	25	2	100
	126	93	69	12	300

Would you say that the sampling techniques adopted by the 2 researches are independent?

Solution:

$\boldsymbol{H_0}$: Data obtained are independent of the sampling techniques adopted by the two researchers.

$\boldsymbol{H_1}$: Data obtained are not independent.

The test statistic is given by

$$\chi^2 = \sum_{i=1}^{r}\sum_{j=1}^{s}\frac{\left(O_{ij}-E_{ij}\right)^2}{E_{ij}} \qquad \chi^2 \; distribution \, with \;\; n = (r-1)(s-1)-1 \quad d.o.f$$

$$E_{ij} = \frac{R_i C_j}{N} \;\; ; \;\; i = 1,2,\ldots r \;\; and \;\; j = 1,2,\ldots.s$$

The expected frequencies are

$$E(86) = \frac{126 * 200}{300} = 84\,; \qquad E(60) = \frac{93 * 200}{300} = 62$$
$$E(44) = \frac{69 * 200}{300} = 46\,; \qquad E(10) = \frac{12 * 200}{300} = 8$$
$$E(40) = \frac{126 * 100}{300} = 42\,; \qquad E(33) = \frac{93 * 100}{300} = 31$$
$$E(25) = \frac{69 * 100}{300} = 23\,; \qquad E(2) = \frac{12 * 100}{300} = 4$$

O_{ij}	E_{ij}	$(O_{ij} - E_{ij})$	$(O_{ij} - E_{ij})^2$	$\frac{(O_{ij} - E_{ij})^2}{E_{ij}}$
86	84	2	4	0.047619
60	62	−2	4	0.064516
44	46	−2	4	0.086957
10	8	2	4	0.5
40	42	−2	4	0.095238
33	31	2	4	0.129032
25	23	2	4	0.173913
2	4	−2	4	1
				2.097275

$$\chi^2 = 2.097$$

Table value of $\chi^2_{0.05}$ with $n = (r-1)(s-1) = (2-1)(4-1) - 1 = 2$ d.o.f is 5.991.

Conclusion:

Since $\chi^2 < \chi^2_{0.05}$, we accept null hypothesis. That is the data are obtained are independent.

Example: 4

Test of fidelity and selectivity of 190 radio receivers produced the results shown in the following table.

Fidelity

Selectivity	**Low**	**Average**	**High**	**Total**
Low	6	12	32	50
Average	33	61	18	112
High	12	15	0	28
Total	52	88	50	190

Use the 0.01 level of significance to test whether there is a relationship between fidelity and selectivity.

Solution:

$\boldsymbol{H_0}$: There is no relationship between fidelity and selectivity.

$\boldsymbol{H_1}$: There is some relationship between fidelity and selectivity.

The test statistic is given by

$$\chi^2 = \sum_{i=1}^{r}\sum_{j=1}^{s}\frac{(O_{ij}-E_{ij})^2}{E_{ij}} \qquad \chi^2\ distribution\ with\ \ n=(r-1)(s-1)-1\ \ d.o.f$$

$$E_{ij} = \frac{R_i C_j}{N}\ ;\ \ i=1,2,\dots r\ \ and\ \ j=1,2,\dots .s$$

The expected frequencies are

$$E(6) = \frac{52*50}{190} = 13.684\,; \qquad E(12) = \frac{88*50}{190} = 23.158$$

$$E(32) = 13.158\,; \qquad E(33) = 30.653$$

$$E(61) = 51.874\,; \qquad E(18) = 29.474$$

$$E(13) = 7.663\,; \qquad E(15) = 12.968; \quad E(0) = 7.368$$

O_{ij}	E_{ij}	$(O_{ij}-E_{ij})$	$(O_{ij}-E_{ij})^2$	$\frac{(O_{ij}-E_{ij})^2}{E_{ij}}$
6	13.684	−7.684	59.04386	4.31481
12	23.158	−11.158	124.501	5.376154
32	13.158	18.842	355.021	26.98138
33	30.653	2.347	5.508409	0.179702
61	51.874	9.126	83.28388	1.605503
18	29.474	−11.474	131.6527	4.466739
13	7.663	5.337	28.48357	3.717026
15	12.968	2.032	4.129024	0.318401
0	7.368	−7.368	54.28742	7.368
				54.32771

$$\chi^2 = 54.33$$

Table value of $\chi^2_{0.01}$ with $n=(r-1)(s-1)=(3-1)(3-1)-1=3$ d.o.f is 11.345.

Conclusion:

Since $\chi^2 > \chi^2_{0.01}$, we reject null hypothesis. That is a relationship between fidelity and selectivity.

Example: 5

A sample of 200 persons with a particular disease was selected. Out of these, 100 were given a drug and the others were not given any drug. The results are as follows.

No. of persons	Drug	No drug	Total
Cured	65	55	120
Not cured	35	45	80
Total	100	100	200

Test whether the drug is effective or not.

The sign test for paired data

Working rule:

1. Omitting zero differences, find the no. of positive deviations in

   $d_i = x_i - y_i, \qquad let\ it\ be\ \ k$

2. Find $p' = P(u \geq k) = \left(\frac{1}{2}\right)^n \sum_{x=k}^{n} \binom{n}{x}$ $\qquad [\because\ \ \mathbf{np} < 5]$

3. If $p' \geq 0.05$, reject H_0 and if $p' > 0.05$ we accept H_0.

Example: 6

A consumer panel includes 14 individuals. It is asked to rate two brands of cococola according to a point evaluation system based on several criteria. The table gives below reports the points assigned. Test the null hypothesis that there is no difference in the level of ratings for the two brands of cococola at 5% LOS using the sign test.

Panel member	Brand I	Brand II
1	20	16
2	24	26
3	28	18
4	24	17
5	20	20

Solution:

H_0: $p = 0.5$, there is no difference in the level of ratings for the two brands.

H_1: $p \neq 0.5$, there is some difference in the level of ratings for the two brands..

From the given data, we have

$$d_i\ :\quad - + - -\ \ 0 - + - + -\ \ 0 - - +$$

Hence $n = 4 + 8 = 12\ \ and\ \ k = 8\ \ (no.of\ negative\ signs)$

The test statistic is given by

$$p' = P(u \geq k) = \left(\frac{1}{2}\right)^n \sum_{x=k}^{n} \binom{n}{x} \qquad [\because\ \ \mathbf{np} < 5]$$

$$= \left(\frac{1}{2}\right)^{12} \sum_{x=8}^{12} \binom{12}{x} = \left(\frac{1}{2}\right)^{12} \left[\binom{12}{8} + \binom{12}{9} + \binom{12}{10} + \binom{12}{11} + \binom{12}{12}\right]$$

$$= 0.000244[495 + 220 + 66 + 12 + 1]$$

$$= 0.1937$$

Conclusion:

Since $p'(0.1937) > 0.05$, we accept null hypothesis and there is no difference in the level of ratings for the two brands.

Example: 7

An automotive engineer is investigating 2 different types of metering devices for an electronic fuel injection system to determine whether they differ in their fuel mileage performance. The system is installed on 12 different cars and a test is run with each metering device on each car. The observed fuel mileage performance data are given in the following table. Use the sign test to determine whether the median fuel mileage performance is the same for both devices using 5% LOS.

Car:	1	2	3	4	5	6	7	8	9	10	11	12
Device I:	17.6	19.4	19.5	17.1	15.3	15.9	16.3	18.4	17.3	19.1	17.8	18.2
Device II:	16.8	20	18.2	16.4	16	15.4	16.5	18	16.4	20.1	16.7	17.9

Solution:

H_0: $p = 0.5$, that is the median fuel mileage performance is the same for both brands.

H_1: $p \neq 0.5$, that is the median fuel mileage performance is not the same for both brands.

From the given data, we have

$$d_i: - + - - + - + - - + - -$$

Hence $n = 4 + 8 = 12$ and $k = 8$ $(no.of\ negative\ signs)$

The test statistic is given by

$$p' = P(u \geq k) = \left(\frac{1}{2}\right)^n \sum_{x=k}^{n} \binom{n}{x} \qquad [\because \ \mathbf{np} < 5]$$

$$= \left(\frac{1}{2}\right)^{12} \sum_{x=8}^{12} \binom{12}{x} = \left(\frac{1}{2}\right)^{12} \left[\binom{12}{8} + \binom{12}{9} + \binom{12}{10} + \binom{12}{11} + \binom{12}{12}\right]$$

$$= 0.000244[495 + 220 + 66 + 12 + 1]$$

$$= 0.1937$$

Conclusion:

Since $p'(0.1937) < 0.05$, we accept null hypothesis and conclude that the median fuel mileage is same for both brands.

Example: 8

The following data shows that the employee's rates of defective work before and after a change in the wage incentive plan. Compare the following two sets of data to see whether the change lowered the defective units produced. Using the sign test with $\propto = 0.01$.

Before	8	7	6	9	7	1	8	6	5	8	10	8
After	6	5	8	6	9	8	10	7	5	6	9	8

Solution:

$H_0:\ p = 0.5$

$H_1:\ p < 0.5$, (one tailed test)

From the given data, we have

$$d_i : - - + - + - + + 0 - - 0$$

Here $n = 4 + 6 = 10\ and\ \ k = 6\ \ (no.of\ negative\ signs)$

The test statistic is given by

$$p' = P(u \geq k) = \left(\frac{1}{2}\right)^n \sum_{x=k}^{n} \binom{n}{x} \qquad [\because\ \text{np} < 5]$$

$$= \left(\frac{1}{2}\right)^{10} \sum_{x=6}^{10} \binom{10}{x} = \left(\frac{1}{2}\right)^{10} \left[\binom{10}{6} + \binom{10}{7} + \binom{10}{8} + \binom{10}{9} + \binom{10}{10}\right]$$

$$= 0.000976[386]$$

$$= 0.3767$$

Conclusion:

Since $p'(0.3767) > 0.05$, we accept null hypothesis and conclude that there is no significant change in the defective units produced.

One sample sign test:

Example: 9

The following data represent the number of hours that a rechargeable hedge trimmer operates before a recharge is required.

1.5 2.2 0.9 1.3 2 1.6 1.8 1.5 2 1.2 1.7

Hypothesis of the 0.05 LOS that this particular trimmer operates with a median of 1.8 hours before requiring a recharge.

Solution:

H_0: $\mu = 1.8$,

H_1: $\mu > 1.8$,

Given data is

1.5 2.2 0.9 1.3 2 1.6 1.8 1.5 2 1.2 1.7

Assign $+$ for greater than 1.8.

Assign $-$ for less than 1.8.

Assign 0 if it is equal to 1.8, we have

$$- + - - + - \ 0 - + - -$$

$n =$ the total no. Of $+$ and – signs.

i.e., $n = 10$

$u =$no. Of plus signs $= 3$

The test statistic is given by

$$p' = P(u \geq 3) = \left(\frac{1}{2}\right)^1 \sum_{x=3}^{10} \binom{10}{x}$$

$$= \left(\frac{1}{2}\right)^{10} \left[\binom{10}{3} + \binom{10}{4} + \binom{10}{5} + \binom{10}{6} + \binom{10}{7} + \binom{10}{8} + \binom{10}{9} + \binom{10}{10}\right]$$

$$= 0.000976[120 + 210 + 252 + 210 + 120 + 45 + 10 + 1]$$

$$p' = 0.9448$$

Conclusion:

Since $p'(0.9448) > 0.05$, we accept null hypothesis and conclude that this particular trimmer operates with a median of 1.8 hours before requiring a recharge.

Example: 10

The following are the measurements of breaking strength of a certain kind of 2-inch cotton ribbon in pounds.

163 165 160 189 161 171 158 151 169 162

163 139 172 165 148 166 172 163 187 173

Solution:

H_0: $\mu = 160$,

H_1: $\mu > 160$,

Given data is

Assign $+$ for greater than 160.

Assign $-$ for less than 160.
Assign 0 if it is equal to 160, we have

$$+ + \ 0 + + + - - + + + - + + - + + + + +$$

$n =$ the total no. Of $+$ and – signs.

i.e., $n = 19$

u =no. Of plus signs $u = 15$

The test statistic is given by

$$p' = P(u \geq 15) = \left(\frac{1}{2}\right)^{19} \sum_{x=15}^{19} \binom{19}{x}$$

$$= \left(\frac{1}{2}\right)^{19} \left[\binom{19}{15} + \binom{19}{16} + \binom{19}{17} + \binom{19}{18} + \binom{19}{19}\right]$$

$$p' = 0.0095$$

Conclusion:

Since $p'(0.0095) < 0.05$, we reject the null hypothesis and conclude that the eman breaking strength of a given kind of ribbon exceeds 160 pounds.

Mann-Whitney U-test

The test statistic is given by

$$Z = \frac{U - \mu}{\sigma} \sim N(0,1)$$

$$Where \quad U = n_1 \, n_2 + \frac{n_1(n_1 + 1)}{2} - R_1$$

$$\mu = \frac{n_1 n_2}{2} \quad and \quad \sigma = \sqrt{\frac{n_1 n_2 (n_1 + n_2 - 1)}{12}}$$

Example: 11

Two methods of instruction to apprentices is to be evaluated. A director assigns 15 randomly selected trainers to each of the two methods. Due to drop outs, 14 complete in batch 1 and 12 complete in batch 2. An achievement test was given to these successful candidates. Their scores are as follows.

Method I	70	90	82	64	86	77	84	79	82	89	73	81	83	66
Method II	86	78	90	82	65	87	80	88	65	85	76	94		

Test whether the two methods have the significant difference in effectiveness. Use Mann-Whitney test at 5% LOS.

Solution:

$H_0 \;:\; \mu_1 = \mu_2$ there is no difference in effectiveness between the two brands.

$H_1 \;:\; \mu_1 \neq \mu_2$ there is some difference in effectiveness between the two brands.

Method I	Rank I	Method II	Rank II
70	4	86	18.5
90	23.5	78	8
82	13	90	23.5
64	1	82	13
86	18.5	65	2
77	7	87	20
84	16	80	10
79	9	88	21
82	13	95	26
89	22	85	17
73	5	76	6
81	11	94	25
83	15		
66	3		
	161		190

$$Here \quad n_1 = 14, \quad n_2 = 12, \quad R_1 = 161, \quad R_2 = 190$$

$$U = n_1\, n_2 + \frac{n_1(n_1+1)}{2} - R_1$$

$$= 14 * 12 + \frac{14(14+1)}{2} - 161$$

$$U = 112$$

$$\mu = \frac{n_1 n_2}{2} = \frac{14 * 12}{2}$$

$$\mu = 84$$

$$\sigma = \sqrt{\frac{n_1 n_2 (n_1 + n_2 - 1)}{12}}$$

$$= \sqrt{\frac{14 * 12(14 + 12 - 1)}{12}} = \sqrt{14 * 27}$$

$$\sigma = 19.4422$$

The test statistic is given by

$$Z = \frac{U - \mu}{\sigma} \sim N(0,1)$$

$$= \frac{112 - 84}{19.4422}$$

$$Z = 1.4402$$

At 5% LOS for two tailed test $Z_{\propto} = 1.96$.

Conclusion:

Since $|Z| < 1.96$, we accept H_0 and conclude that there is no difference in effectiveness between two methods.

Example: 12

The nicotine content of two brands of cigarettes measured in milligrams was found to be as follows.

Brand A	2.1	4	6.3	5.4	4.8	3.7	6.1	3.3		
Brand B	4.1	0.6	3.1	2.5	4	6.2	1.6	2.2	1.9	5.4

Use the rank-sum test; test the hypothesis, at 0.05 LOS, that the average nicotine contents of the two brands are equal against the alternative that they are equal.

Solution:

H_0 : $\mu_1 = \mu_2$ the average nicotine content are equal.

H_1 : $\mu_1 \neq \mu_2$ the average nicotine content are not equal.

$Bran\ A$	$Rank\ R_1$	$Brand\ B$	$Rank\ R_2$
2.1	4	4.1	12
4	10.5	0.6	1
6.3	18	3.1	7
5.4	14.5	2.5	6
4.8	13	4.6	10.5
3.7	9	6.2	17
6.1	16	1.6	2
3.3	8	1.9	3
		5.4	14.5
	93		73

$$Here \quad n_1 = 8, \quad n_2 = 10, \quad R_1 = 93, \quad R_2 = 73$$

$$U = n_1\, n_2 + \frac{n_1(n_1 + 1)}{2} - R_1$$

$$= 8 * 10 + \frac{8(8+1)}{2} - 93$$

$$U = 23$$

$$\mu = \frac{n_1 n_2}{2} = \frac{8 * 10}{2}$$

$$\mu = 40$$

$$\sigma = \sqrt{\frac{n_1 n_2 (n_1 + n_2 - 1)}{12}}$$

$$= \sqrt{\frac{8 * 10(8 + 10 - 1)}{12}} = \sqrt{\frac{1520}{12}}$$

$$\sigma = 11.2546$$

The test statistic is given by

$$Z = \frac{U - \mu}{\sigma} \sim N(0,1)$$

$$= \frac{23 - 40}{11.2546}$$

$$Z = 1.5105$$

At 5% LOS for two tailed test $Z_{\propto} = 1.96$.

Conclusion:

Since $|Z| < 1.96$, we accept H_0 and the average nicotine content of two brands are equal.

Example: 13

Twelve children one each selected from 12 sets of identical twins, were trained by a certain method A and the remaining 12 children were trained by method B. at the end of the year, the following I.Q scores were obtained.

Pair	1	2	3	4	5	6	7	8	9	10	11	12
Method A	124	118	127	120	135	130	140	128	140	126	130	126
Method B	131	127	135	128	137	131	132	125	141	118	132	129

Is this sufficient evidence to indicate a difference in the average IQ scores of the two groups?

Solution:

$H_0 \;:\; \mu_1 = \mu_2$.

$H_1 \;:\; \mu_1 \neq \mu_2$.

$Method\ A$	$Rank\ R_1$	$Method\ B$	$Rank\ R_2$
124	4	131	15.5
118	1.5	127	8.5
127	8.5	135	19.5
120	3	128	10.5

135	19.5	137	21
130	13.5	131	15.5
140	22.5	132	17.5
128	10.5	125	5
140	22.5	141	24
126	6.5	118	1.5
130	13.5	132	17.5
126	6.5	129	12
	132		168

$$Here \quad n_1 = 12, \quad n_2 = 12, \quad R_1 = 132, \quad R_2 = 168$$

$$U = n_1\, n_2 + \frac{n_1(n_1+1)}{2} - R_1$$

$$= 12*12 + \frac{12(12+1)}{2} - 132$$

$$U = 90$$

$$\mu = \frac{n_1 n_2}{2} = \frac{12*12}{2}$$

$$\mu = 72$$

$$\sigma = \sqrt{\frac{n_1 n_2(n_1+n_2-1)}{12}}$$

$$= \sqrt{\frac{12*12(12+12-1)}{12}} = \sqrt{12*25}$$

$$\sigma = 17.3205$$

The test statistic is given by

$$Z = \frac{U-\mu}{\sigma} \sim N(0,1)$$

$$= \frac{90-72}{17.3205}$$

$$Z = 1.0932$$

At 5% LOS for two tailed test $Z_\propto = 1.96$.

Conclusion:

Since $|Z| < 1.96$, we accept H_0. There is no sufficient evidence to indicate a difference in the average IQ scores of the two groups.

Example: 14

The following random samples are measurements of the heat producing capacity (in millions of calories per ton) of specimens of coal from the two mines.

Mine I	31	25	38	33	42	40	44	26	43	35
Mine II	44	30	34	47	35	32	35	47	48	34

Test the hypothesis of no difference between the mine I and mine II. Using the Mann-Whitney test for the above sample data at 0.10 LOS.

Solution:

$H_0 \;:\; \mu_1 = \mu_2$.

$H_1 \;:\; \mu_1 \neq \mu_2$.

$Mine\ I$	$Rank\ R_1$	$Mine\ II$	$Rank\ R_2$
31	4	44	16.5
25	1	30	3
38	12	34	7.5
33	6	47	18.5
42	14	35	10
40	13	32	5
44	16.5	35	10
26	12	47	18.5
43	15	48	20
35	10	34	7.5
	103.5		116.5

$$Here \quad n_1 = 10, \quad n_2 = 10, \quad R_1 = 103.5, \quad R_2 = 116.5$$

$$U = n_1\, n_2 + \frac{n_1(n_1+1)}{2} - R_1$$

$$= 10 * 10 + \frac{10(10+1)}{2} - 103.5$$

$$U = 51.5$$

$$\mu = \frac{n_1 n_2}{2} = \frac{10 * 10}{2}$$

$$\mu = 50$$

$$\sigma = \sqrt{\frac{n_1 n_2 (n_1 + n_2 - 1)}{12}}$$

$$= \sqrt{\frac{10 * 10(10 + 10 - 1)}{12}}$$

$$\sigma = 13.2288$$

The test statistic is given by

$$Z = \frac{U - \mu}{\sigma} \sim N(0,1)$$

$$= \frac{51.5 - 50}{13.2288}$$

$$Z = 0.11389$$

At 1% LOS for two tailed test $Z_{\propto} = 1.645$.

Conclusion:

Since $|Z| < 1.645$, we accept H_0. There is no difference between mine I and mine II.

Kruskal-Wallis test or H-test

Null hypothesis $H_0 \ : \ \mu_1 = \mu_2 = \mu_3,$

Null hypothesis $H_1 \ : \ \mu_1 \neq \mu_2 \neq \mu_3,$

The test statistic is given by

$$W = \frac{12}{n(n+1)}\left[\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \frac{R_3^2}{n_3}\right] - 3(n+1)$$

where $n_i =$ the no. of items in sample i

$k =$ no. of populations (or samples)

$n = n_1 + n_2 + \ \ldots\ldots + n_i$

R_i =sum of the ranks of all items in a sample i

If H falls in the critical region $H < \chi^2_\alpha$ with $(k-1)$ d.o.f, we accept our null hypothesis.

Example:

Use Kruskal-Wallis test to test for difference in mean among the 3 samples. If $\propto = 0.01$, what are your conclusions.

Sample I	95	97	99	98	99	99	99	94	95	98
Sample II	104	102	102	105	99	102	111	103	100	103
Sample III	119	130	132	136	141	172	145	150	144	135

Solution:

$H_0 \ : \ \mu_1 = \mu_2 = \mu_3,$ there is no difference in mean among three samples.

$H_1 \ : \ \mu_1 \neq \mu_2 \neq \mu_3,$ there is some difference in mean among three samples.

$Sample\ I$	$Rank\ R_1$	$Sample\ II$	$Rank\ R_2$	$Sample\ III$	$Rank\ R_3$
95	2.5	104	18	119	21
97	4	102	14	180	22
99	9	102	14	132	23
98	5.5	105	19	136	25
99	9	99	9	141	26
99	9	102	14	172	30
99	9	111	20	145	28
94	1	103	16.5	150	29

95	2.5	100	12	144	27
98	5.5	103	16.5	135	24
	$\boldsymbol{R_1 = 57}$		$\boldsymbol{R_2 = 153}$		$\boldsymbol{R_3 = 255}$

$$n = n_1 + n_2 + n_3 = 30$$

The test statistic is given by

$$W = \frac{12}{n(n+1)}\left[\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \frac{R_3^2}{n_3}\right] - 3(n+1)$$

$$= \frac{12}{30(30+1)}\left[\frac{57^2}{10} + \frac{153^2}{10} + \frac{255^2}{10}\right] - 3(30+1)$$

$$= 0.0129\,[9168.3] - 93$$

$$W = 25.27$$

The χ^2 value at 1% LOS with 2 d.o.f is 9.21.

Conclusion:

Since $W > \chi^2_\alpha\ (9.21)$, we reject H_0. There is a significant difference between three sample means.

Example: 15

A company's trainees are randomly assigned to two groups which are taught a certain industrial inspection procedure by 3-different methods. At the end of the instruction period they are tested for inspection performance quality. The following are their scores.

Method A	80	83	79	85	90	68	
Method B	82	84	60	72	86	67	91
Method C	93	65	77	78	88		

Use H-test to determine at the 0.05 LOS whether the 3-methods are equally effective.

Solution:

$H_0\ :\ \mu_1 = \mu_2 = \mu_3,$ the 3-methods are equally effective.

$H_1\ :\ \mu_1 \neq \mu_2 \neq \mu_3,$ the 3-methods are not equally effective.

$Method\ A$	R_1	$Method\ B$	R_2	$Method\ C$	R_3
80	9	82	10	93	18
83	11	84	12	65	2
79	8	60	1	77	6
85	13	72	5	78	7
90	16	86	14	88	15
68	4	67	3		

		91	17		
	$R_1 = 61$		$R_2 = 62$		$R_3 = 48$

$$n = n_1 + n_2 + n_3 = 6 + 7 + 5 = 18$$

The test statistic is given by

$$W = \frac{12}{n(n+1)}\left[\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \frac{R_3^2}{n_3}\right] - 3(n+1)$$

$$= \frac{12}{18(18+1)}\left[\frac{61^2}{6} + \frac{62^2}{7} + \frac{48^2}{5}\right] - 3(18+1)$$

$$= 0.0351\,[1630.11] - 57$$

$$W = 0.2168$$

The χ^2 value at 5% LOS with $k-1 = 3-1 = 2$ d.o.f is 5.991.

Conclusion:

Since $W < \chi_\alpha^2\ (5.991)$, we accept H_0. That is the three methods are equally effective.

Example: 16

An information systems company investigated the computer literacy of managers. As a part of their study, the company designed a questionnaire. To check the design of the questionnaire (i.e., its validity), 19 managers were randomly selected and asked to complete the questionnaire. The managers were classified as A,B and C based on their knowledge and experience. The scores appear in the table below. Is there sufficient evidence to conclude that the mean scores differ for the 3-groups of managers? Use $\propto = 0.05$.

Method A	80	83	79	85	90	68	
Method B	82	84	60	72	86	67	91
Method C	93	65	77	78	88		

Use H-test to determine at the 0.05 LOS whether the 3-methods are equally effective.

Example: 17

A quality control engineer in an electronics plant has sampled the output of three assembly lines and recorded the number of defects observed. The samples involve the entire output of the three lines for 10 randomly selected hours from a given week. Do the data provide sufficient evidence to indicate that at least one of the line tends to produce more defects than the others. Test at 5% LOS using suitable non-parametric test.

Line 1	6	38	3	17	11	30	15	16	25	5
Line 2	34	28	42	13	40	31	9	32	39	27
Line 3	13	35	19	4	29	0	7	33	18	24

Solution:

H_0 : $\mu_1 = \mu_2 = \mu_3$, the 3-methods are equally effective.

H_1 : $\mu_1 \neq \mu_2 \neq \mu_3$, the 3-methods are not equally effective.

Line I	Rank R_1	Line II	Rank R_2	Line III	Rank R_3
6	4	34	24	13	8.5
38	26	28	18	35	25
3	1	42	29	19	14
17	12	13	8.5	4	2
11	7	40	28	29	19
30	20	31	21	0	0
15	10	9	6	7	5
16	11	32	22	33	23
25	16	39	27	18	13
5	3	27	19	24	15
	$R_1 = 110$		$R_2 = 202.5$		$R_3 = 124.5$

$$n_1 = 10,\ n_2 = 10,\ n_3 = 9$$
$$n = n_1 + n_2 + n_3 = 10 + 10 + 9 = 29$$

The test statistic is given by

$$W = \frac{12}{n(n+1)}\left[\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \frac{R_3^2}{n_3}\right] - 3(n+1)$$
$$= \frac{12}{29(29+1)}\left[\frac{110^2}{10} + \frac{202.5^2}{10} + \frac{124.5^2}{9}\right] - 3(29+1)$$

The χ^2 value at 5% LOS with $k - 1 = 3 - 1 = 2$ d.o.f is 5.991.

Conclusion:

Since $W < \chi^2_\alpha\ (5.991)$,

Example: 18

A research company has designed three different systems to clean up oil spills. The following table contains the results, measured by how much surface area (I square meters) is cleaned in one hour. The data were found by testing each method in several trails. Are three systems equally effective? Use 5% LOS.

System A	55	60	63	56	59	55
System B	57	53	64	49	62	
System C	66	52	61	57		

Solution:

H_0 : $\mu_1 = \mu_2 = \mu_3$, the 3-systems are equally effective.

H_1 : $\mu_1 \neq \mu_2 \neq \mu_3$, the 3-systems are not equally effective.

$System\ A$	R_1	$System\ B$	R_2	$System\ C$	R_3
55	4.5	57	7.5	66	15
60	10	53	3	52	2
63	13	64	14	61	11
56	6	49	1	57	7.5
59	9	62	12		
55	4.5				
	$R_1 = 47$		$R_2 = 37.5$		$R_3 = 35.5$

$$n = n_1 + n_2 + n_3 = 6 + 5 + 4 = 15$$

The test statistic is given by

$$W = \frac{12}{n(n+1)}\left[\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \frac{R_3^2}{n_3}\right] - 3(n+1)$$

$$= \frac{12}{15(15+1)}\left[\frac{47^2}{6} + \frac{37.5^2}{5} + \frac{35.5^2}{4}\right] - 3(15+1)$$

$$= 0.05\,[964.4792] - 48$$

$$W = 0.224$$

The χ^2 value at 5% LOS with $k-1 = 3-1 = 2$ d.o.f is 5.991.

Conclusion:

Since $H < \chi^2_\alpha\ (5.991)$, we accept H_0. That is the three systems are equally effective.

Example: 19

Three different brands of king-size cigarettes were tested for tar content in a pack of 10 cigarettes. The tar content in milligram for the three brands is found as in the following table. Using Kruskal-Wallis test, verify that $\propto = 0.05$ LOS that there is no significant difference in the three brands of cigarettes in terms of the tar content.

	X	**Y**	**Z**
1	10	16	12
2	14	13	14
3	13	11	10
4	11	14	17
5	12	10	11

Solution:

$H_0\ :\ \mu_1 = \mu_2 = \mu_3,$ the 3-systems are equally effective.

$H_1\ :\ \mu_1 \neq \mu_2 \neq \mu_3,$ the 3-systems are not equally effective.

$Brand - X$	R_1	$Brand - Y$	R_2	$Brand - Z$	R_3

10	2	16	14	12	7.5
14	12	13	9.5	14	12
13	9.5	11	5	10	2
11	5	14	12	17	15
12	7.5	10	2	11	5
	$R_1 = 36$		$R_2 = 42.5$		$R_3 = 41.5$

$$n = n_1 + n_2 + n_3 = 5 + 5 + 5 = 15$$

The test statistic is given by

$$W = \frac{12}{n(n+1)}\left[\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \frac{R_3^2}{n_3}\right] - 3(n+1)$$

$$= \frac{12}{15(15+1)}\left[\frac{36^2}{5} + \frac{42.5^2}{5} + \frac{41.5^2}{5}\right] - 3(15+1)$$

$$= 0.05\,[964.9] - 48$$

$$W = 0.245$$

The χ^2 value at 5% LOS with $k - 1 = 3 - 1 = 2$ d.o.f is 5.991.

Conclusion:

Since $H < \chi^2_\alpha\,(5.991)$, we accept H_0. That is the three brands of cigarettes in terms of tar content are equal.

One sample run test:

The test statistic is given by

$$z = \frac{R - \mu}{\sigma} \sim N(0,\ 1)$$

$$where \quad \mu = \frac{2\,n_1 n_2}{n_1 + n_2} + 1 \quad and \quad \sigma = \sqrt{\frac{2\,n_1\,n_2\,\left(2\,n_1\,n_2 - n_1 - n_2\right)}{(n_1 + n_2)^2\,\left(n_1 + n_2 - 1\right)}}$$

R is the number of runs.

If $|Z| \le 1.96$ accept H_0, at 5% level, otherwise reject H_0.

Example: 20

A technician is asked to analyze the results of 22 items made in a preparation run. Each item has been measured and compared to engineering specifications. The order of acceptance 'a' and rejections 'r' is

a a r r r a r r a a a a a r r a r r a a r a

Determine whether it is a random sample or not. Use $\propto = 0.05$.

Solution:

Given

$$\frac{aa}{1}\ \frac{rrr}{2}\ \frac{a}{3}\ \frac{rr}{4}\ \frac{aaaaa}{5}\ \frac{rr}{6}\ \frac{a}{7}\ \frac{rr}{8}\ \frac{aa}{9}\ \frac{r}{10}\ \frac{a}{11}$$

$Here \quad n_1 = 12 \ (a), \quad and \quad n_2 = 10 \ (r)$

$$R = 11 \qquad (the\ no.of\ runs\ )$$

$$\mu = \frac{2\,n_1 n_2}{n_1 + n_2} + 1$$

$$= \frac{2 * 12 * 10}{12 + 10} + 1 = \frac{240}{22} + 1$$

$$\mu = 11.9091$$

$$\sigma = \sqrt{\frac{2\,n_1\,n_2\ (2\,n_1\,n_2 - n_1 - n_2)}{(n_1 + n_2)^2\ (n_1 + n_2 - 1)}}$$

$$= \sqrt{\frac{2 * 12 * 10\ (2 * 12 * 10 - 12 - 10)}{(12 + 10)^2 (12 + 10 - 1)}}$$

$$= \sqrt{\frac{52320}{10164}}$$

$$\sigma = 2.2688$$

H_0 : The sample is randomly chosen

H_1 : The sample is not randomly chosen

The test statistic is given by

$$z = \frac{R - \mu}{\sigma} \sim N(0,\ 1)$$

$$Z = \frac{11 - 11.9091}{2.2688}$$

$$Z = -4.007$$

$$\therefore\ |Z| = 0.4007$$

The value of $Z_\propto$ at $\propto = 0.05$ LOS for two tailed test is 1.96.

Conclusion:

Since $|Z| < 1.96$, we accept the null hypothesis. That is sample is randomly chosen.

Example: 21

In an industrial production line items are inspected periodically for defectives. The following is a sequence of defective items (D) and non-defective items (N) produced by these production line.

DD NNN D NN DD NNNNN DDD NN D NNNN D N D

Test whether the defectives are occurring at random or not at 5% LOS.

Solution:

Given

$$\frac{DD}{1}\quad \frac{NNN}{2}\quad \frac{D}{3}\quad \frac{NN}{4}\quad \frac{DD}{5}\quad \frac{NNNNN}{6}\quad \frac{DDD}{7}\quad \frac{NN}{8}\quad \frac{D}{9}\quad \frac{NNNN}{10}\quad \frac{D}{11}\quad \frac{N}{12}\quad \frac{D}{13}$$

$Here \quad n_1 = 11 \ \ (D), \quad and \quad n_2 = 17 \ \ (N)$

$$R = 13 \qquad (the\ no.of\ runs\ )$$

$$\mu = \frac{2\, n_1 n_2}{n_1 + n_2} + 1$$

$$= \frac{2 * 11 * 17}{11 + 17} + 1$$

$$\mu = 14.357$$

$$\sigma = \sqrt{\frac{2\, n_1\, n_2\ (2\, n_1\, n_2 - n_1 - n_2)}{(n_1 + n_2)^2\ (n_1 + n_2 - 1)}}$$

$$= \sqrt{\frac{2 * 11 * 17\ (2 * 11 * 17 - 11 - 17)}{(11 + 17)^2 (11 + 17 - 1)}}$$

$$= \sqrt{\frac{129404}{21168}} = \sqrt{6.113}$$

$$\sigma = 2.472$$

H_0 : Defectives occur at random

H_1 : Defectives not occur at random

The test statistic is given by

$$z = \frac{R - \mu}{\sigma} \ \sim\ N(0,\ \ 1)$$

$$Z = \frac{13 - 14.357}{2.472}$$

$$Z = -0.5489$$

$$\therefore\ \ |Z| = 0.5489$$

The value of $Z_\propto$ at $\propto = 0.05$ LOS for two tailed test is 1.96.

Conclusion:

Since $|Z| < 1.96$, we accept H_0. The defectives occur at random.

Example: 22

In 30 tosses of a coin the following sequence of heads (H) and tails (T) is obtained.

H TT J T H H H T H H T T T H T H T H H T H T T T H T H H T H T

a) Determine the number of runs.

b) Test at the 5% LOS whether the sequence is random.

Solution:

Given

H	TT	H	T	HHH	T	HH	TT	H	T	H	T	HH	T	H	TT	H	T	HH	T	H	T
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22

$Here \quad n_1 = 16 \;\; (H), \quad and \quad n_2 = 14 \;\; (T)$

$$R = 22 \qquad (the\ no.\ of\ runs\ )$$

$$\mu = \frac{2\, n_1 n_2}{n_1 + n_2} + 1$$

$$= \frac{2 * 16 * 14}{16 + 14} + 1$$

$$\mu = 15.9333$$

$$\sigma = \sqrt{\frac{2\, n_1\, n_2\ (2\, n_1\, n_2 - n_1 - n_2)}{(n_1 + n_2)^2\ (n_1 + n_2 - 1)}}$$

$$= \sqrt{\frac{2 * 16 * 14\ (2 * 16 * 14 - 16 - 14)}{(16 + 14)^2 (16 + 14 - 1)}}$$

$$= \sqrt{\frac{187264}{26100}}$$

$$\sigma = 2.6786$$

H_0 : The sample is randomly chosen

H_1 : The sample is not randomly chosen

The test statistic is given by

$$z = \frac{R - \mu}{\sigma} \quad \sim \ N(0,\ 1)$$

$$Z = \frac{22 - 15.933}{2.6786}$$

$$Z = 2.2649$$

The value of $Z_\propto$ at $\propto = 0.10$ LOS for two tailed test is 1.645.

Conclusion:

Since $|Z| > 1.645$, we reject H_0. The sample is not randomly chosen.

Rank correlation

The spearman's coefficient of rank correlation is given by

$$\rho_s = 1 - \left[\frac{6 \sum d^2}{n(n^2 - 1)}\right] \quad where \quad d_i = x_i - y_i$$

The rank correlation ρ lies between $-1 \le \rho < +1$.

Repeated ranks:

$$\rho_s = 1 - \left\{\frac{6[\sum d^2 + C.F_1 + C.F_2 + \cdots..]}{n(n^2-1)}\right\}$$

$$where\ C.F = \frac{m(m^2-1)}{12}$$

Example: 23

The following are the ranks obtained by 10 students in statistics and mathematics. Test what extent is knowledge of students in statistics related to knowledge in mathematics?

Statistics	1	2	3	4	5	6	7	8	9	10
Mathematics	2	4	1	5	3	9	7	10	6	8

Solution:

Rank in Stat	Rank in Maths	$d = x_i - y_i$	d^2
1	2	-1	1
2	4	-2	4
3	1	2	4
4	5	-1	1
5	3	2	4
6	9	-3	9
7	7	0	0
8	10	-2	4
9	6	3	9
10	8	2	4
			40

$$\therefore \quad \rho_s = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6 * 40}{10(10^2-1)} = 0.76$$

There is high correlation between knowledge in the two subjects.

Example: 24

Ten competitions in a beauty contest are ranked by 3 judges in the following order.

A	1	6	5	3	10	2	4	9	7	8
B	3	5	8	4	7	10	2	1	6	9
C	6	4	9	8	1	2	3	10	5	7

Find which pair of judges has the nearest approach to common taste of beauty.

Solution:

A	B	C	$d_1 = A - B$	$d_2 = B - C$	$d_3 = A - C$	d_1^2	d_2^2	d_3^2

1	3	6	-2	-3	-5	4	9	25
6	5	4	1	1	2	1	1	4
5	8	9	-3	-1	-4	9	1	16
3	4	8	-1	-4	-5	1	16	25
10	7	1	3	6	9	9	36	81
2	10	2	-8	8	0	64	64	0
4	2	3	2	-1	1	4	1	1
9	1	10	8	-9	-1	64	81	1
7	6	5	1	1	2	1	1	4
8	9	7	-1	2	1	1	4	1
						$\sum d_1{}^2 = 200$	$\sum d_1{}^2 = 214$	$\sum d_1{}^2 = 60$

$$\therefore \quad \rho_{AB} = 1 - \frac{6\sum d_1{}^2}{n(n^2-1)} = 1 - \frac{6 * 200}{10(10^2-1)} = -0.212$$

$$\therefore \quad \rho_{BC} = 1 - \frac{6\sum d_2{}^2}{n(n^2-1)} = 1 - \frac{6 * 214}{10(10^2-1)} = -0.297$$

$$\therefore \quad \rho_{AC} = 1 - \frac{6\sum d_3{}^2}{n(n^2-1)} = 1 - \frac{6 * 60}{10(10^2-1)} = 0.636$$

Hence judges A and C have the nearest approach to common tastes of beauty.

Example: 25

Calculate the coefficients of rank correlation from the following data.

X:	48	34	40	12	16	16	66	25	16	57
Y:	15	15	24	8	13	6	20	9	9	15

Solution:

X	Y	$ran\ X$	$rank\ Y$	$d = x_i - y_i$	d^2
48	15	3	4	-1	1
34	15	5	4	1	1
40	24	4	1	3	9
12	8	10	9	1	1
16	13	8	6	2	4
16	6	8	10	-2	4
66	20	1	2	-1	1
25	9	6	7.5	-1.5	2.25
16	9	8	7.5	0.5	0.25
57	15	2	4	-2	4
					27.5

$$\therefore \quad \rho_s = 1 - \frac{6[\sum d^2 + C.F_1 + C.F_2 + \cdots \ldots]}{n(n^2-1)}$$

$$Where \quad C.F_1 = \frac{m(m^2-1)}{12}$$

In X −sseries the value 16 is repeated three times, we have

$$C.F_1 = \frac{3(3^2-1)}{12} = 2$$

In Y −sseries the value 15 is repeated three times, and 9 is repeated two times, we have

we have

$$C.F_2 = \frac{3(3^2-1)}{12} = 2 \quad and \quad C.F_3 = \frac{2(2^2-1)}{12} = \frac{1}{2} = 0.5$$

$$\therefore \quad \rho_s = 1 - \frac{6[27.5+2+2+0.5]}{10(10^2-1)} = 1 - \frac{192}{990}$$

$$\rho_s = 0.0860$$

There is a high positive correlation.

Test for Rank correlation Coefficient

The test statistic is given by

$$Z = \frac{r_s - 0}{\frac{1}{\sqrt{n-1}}} = r_s\left[\sqrt{n-1}\right] \sim N(0,1)$$

If $|Z| \leq Z_\propto$, we accept H_0, otherwise reject H_0.

Example: 26

The following are the year of experience (X) and the average customer satisfaction (Y) for 10 service providers. Is there a significant rank correlation between 2 measures? Use the 5% LOS.

X	6.3	5.8	6.1	6.9	3.4	1.8	9.4	4.7	7.2	2.4
Y	5.3	8.6	4.7	4.2	4.9	6.1	5.1	6.3	6.8	5.2

Solution:

H_0: $\rho_s = 0$. That is there is no significant rank correlation between the two measures.

H_1: $\rho_s \neq 0$. That is there is a significant rank correlation between the two measures.

The test is given by

X	R_1	Y	R_2	$d = R_1 - R_2$	d^2
6.3	4	5.3	5	-1	1
5.8	6	8.6	1	5	25
6.1	5	4.7	9	-4	16
6.9	3	4.2	10	-7	49
3.4	8	4.9	8	0	0
1.8	10	6.1	4	6	36
9.4	1	5.1	7	-6	36
4.7	7	6.3	3	4	16
7.2	2	6.8	2	0	0

2.4	9	5.2	6	3	9
					188

The sample rank correlation coefficient

$$\rho_s = 1 - \frac{6\sum d^2}{n(n^2-1)}$$

$$= 1 - \frac{6 * 188}{10 * 99}$$

$$\rho_s = -0.139$$

The expected or critical value at 5% level of significance with $n = 10$ is 0.634.

Conclusion:

Since $|\rho_s| < 0.6364$, we accept H_0 and conclude that there is no significant rank correlation between the two measures.

Example: 27

A consumer panel tested 9 ranks microwave ovens for overall quality. The ranks assigned by the panel and the suggested retail were as follows

Manufacturers	:	1	2	3	4	5	6	7	8	9
Panel rating	:	6	9	2	8	5	1	7	4	2
Suggested price	:	480	395	575	550	510	545	400	465	420

Is there a significant relationship between the quality and the price of a microwave oven? Use 5% LOS.

Solution:

X	R_1	Y	R_2	$d = R_1 - R_2$	d^2
6	6	480	5	1	1
9	9	395	1	8	64
2	2	575	9	-7	49
8	8	550	8	0	0
5	5	510	6	-1	1
1	1	545	7	-6	36
7	7	400	2	5	25
4	4	465	4	0	0
3	3	420	5	-2	4
					180

The sample rank correlation coefficient

$$r_s = 1 - \frac{6\sum d^2}{n(n^2-1)}$$

$$= 1 - \frac{6 * 180}{9(9^2-1)}$$

$$r_s = -0.5$$

$$|r_s| = 0.5$$

$$H_0 \;:\; \rho_s = 0$$
$$H_0 \;:\; \rho_s \neq 0$$

The expected or critical value at 5% level of significance with $n = 9$ is 0.6833.

Conclusion:

Since $|r_s| < 0.6833$, we accept H_0 and conclude that there is no significant rank correlation between the quality and the price of a microwave oven.

Example: 28

The following are ratings aggressiveness (X) and amount of sales (Y) in the last year for eight salespeople. Is there a significant different rank correlation between the two measures? Use the 0.10 LOS.

X	30	17	35	28	42	25	19	29
Y	35	31	43	46	50	32	33	42

Solution:

X	R_1	Y	R_2	$d = R_1 - R_2$	d^2
30	6	35	4	2	4
17	1	31	1	0	0
35	7	43	6	1	1
28	4	46	7	-3	9
42	8	50	8	0	0
25	3	32	2	1	1
19	2	33	3	-1	1
29	5	42	5	0	0
					16

The sample rank correlation coefficient

$$r_s = 1 - \frac{6\sum d^2}{n(n^2-1)}$$
$$= 1 - \frac{6 * 16}{8(8^2-1)}$$
$$r_s = 0.8095$$

$$H_0 \;:\; \rho_s = 0$$
$$H_0 \;:\; \rho_s \neq 0$$

The expected or critical value at 1% level of significance with $n = 8$ is 0.619.

Conclusion:

Since $|r_s| > 0.619$, we reject H_0 and conclude that there is no significant rank correlation between the two measures.