SRI RAMAKRISHNA INSTITUTE OF TECHNOLOGY

DEPARTMENT OF SCIENCE & HUMANITIES

STATISTICS & NUMERICAL METHODS

TWO MARKS

UNIT-I HYPOTHESIS TESTING

1. What are the applications of t distributions?

* Test the hypothesis about the population mean for small samples

* Test the hypothesis about the difference between two means for small samples.

2. What are the conditions for the validity of ψ^2 test?

Solution:

1. The experimental data (samples) must be independent to each other.
2. The total frequency (no. of observations in the sample) must be large, say ≥ 50.
3. All the individual data's should be greater than 5.
4. The no. of classes n must lies in $4 \leq n \leq 16$.

3. What are the parameters and statistics in sampling?

Solution:

To avoid verbal confusion with the statistical constant of the population, namely $\boldsymbol{mean}\,\mu$, $variance\ \sigma^2$ which are usually referred to as parameters. Statistical measures computed from sample observations alone.

E.g. $\boldsymbol{mean}\ (\bar{x}), variance\ (s^2),\ etc.$ are usually referred to as statistic.

4. Define standard error.

Solution:

The standard deviation of a sampling distribution of a statistic is known as its standard error.

5. Define Null Hypothesis.

Solution:

For applying the test of significance, we first set up of a hypothesis – a definite statement about the population parameter. Such a hypothesis is usually a hypothesis of no difference and it is denoted by H_0.

6. What is Type-I and Type-II error?

Solution:

(i). Type-I error : Reject H_0 when it is true.

(ii). Type-II error : Accept H_0 when it is wrong.

7. **Define test statistics.**

$$z = \frac{t - E(t)}{s\,E(t)}$$

8. **Write the application of $'F'$ test and ψ^2 test.**

Solution:

$'F'$ ***test*** **:** To test if the 2 samples have come from the same population.

$'\psi^{2}{}'$ ***test*** **:** To test if the significance of similarity between experimental values and the theoretical values.

9. **Write any two applications of $'\psi^2\,'$ test.**

$'\psi^{2}{}'$ $test$ is used to test whether differences between observed and expected frequencies or significances.

10. **Mention various steps involved in testing of hypothesis.**

Solution:

(i). Set up the null hypothesis.

(ii). Choose the appropriate level of significance (e.g 5% or 1% or 10%)

(iii). Calculate the test statistic $z = \frac{t-E(t)}{s\,E(t)}$.

(iv). Draw conclusion. If $|Calc\ value| < Table\ vale$ then Accept H_0.

If $|Calc\ value| > Table\ vale$ then Reject H_0.

11. **Define Chi-square test for goodness of fit.**

Solution:

Karl Pearson developed a test for testing the significance of similarity between experimental values and the theoretical values obtained under some theory or hypothesis. This test is known as $\boldsymbol{\psi^2}$ test of goodness of fit. Karl Pearson proved that the statistic

$$\boldsymbol{\psi^2 = \sum \frac{(O-E)^2}{E},\ \ where\ \ O - Observed\ frequency,\ \ E - Expected\ frequency}$$

$\boldsymbol{\psi^2}$ is used to test whether differences between observed and expected frequencies are significant.

12. **What are the parameters and statistics in sampling?**

Solution:

To avoid verbal confusion with the statistical constant of the population, namely $\boldsymbol{mean}\ \mu,\ \ variance\ \sigma^2$ which are usually referred to as parameters. Statistical measures computed from sample observations alone. E.g. $\boldsymbol{mean}\ (\bar{x}), variance\ (s^2),\ etc.$ are usually referred to as statistic.

13. **Define level of significance.**

The probability $'\alpha'$ that a random value of the statistic 't' belongs to the critical region is known as the level of significance.

In other words, level of significance is the size of the Type I error. The levels of significance usually employed in testing of hypothesis are 5% and 1%.

UNIT-II. DESIGN OF EXPERIMENTS

1. Define Analysis of variance.

Analysis of variance (ANOVA) is a technique that will enable us to test for the significance of the difference among more than two sample means.

2. What are the assumptions in analysis of variance?

1. Each of the samples is drawn from a normal population.
2. The variances for the population from which samples have been drawn are equal.
3. The variation of each value around its own grand mean should be independent for each value.

3. What are the three essential steps to plan an experiment?

Solution:

1. A statement of the objective. Statement should clearly mention the hypothesis to be tested.
2. A description of the experiment. Description should include the type of experimental material, size of the experiment and the number of replications.
3. The outline of the method of analysis. The outline of method consists of analysis of variance.

4. What are the basic steps in ANOVA?

i. One estimate of the population variance from the variance.

ii. Determine a second estimate of the population variance form the variance with in the sample.

iii. Compare these two estimates if they are approximately equal in value, accept the null hypothesis.

5. Write the steps to find F-ratio.

Solution:

$$F = \frac{Between\ Coloumn\ variance}{With\ in\ Coloumn\ variance} = \frac{Variance\ between\ samples}{Variance\ within\ samples} = \frac{S_1^2}{S_2^2}$$

6. Write down the ANOVA table for One-way classification.

Solution:

Source of variation	Sum of squares	Degrees of freedom	Mean sum of square	F-ratio
Between samples	SSC	$v_1 = K - 1$	$MSC = \frac{SSC}{K-1}$	$F_c = \frac{MSC}{MSC}$
Within samples	SSE	$v_2 = N - K$	$MSE = \frac{SSE}{N-K}$	

7. What are the advantages of completely randomized block design?

Solution:

(i). Easy to lay out.

(ii). Allows flexibility

(iii). Simple statistical analysis.

(iv). The lots of information due to missing data is smaller than with any other design.

8. State the uses of ANOVA.

Solution:

Analysis of variance is useful, for example, for determining (i) which of the various training methods produces the fastest learning record, (ii) whether the effects of some fertilizers on the yields are significantly different (iii) whether the mean qualities of outputs of various machines differ significantly, etc.

9. How would you compare the calculated value F_c $with$ F_T and conclude the analysis of variance?

Solution:

If the computed value of F −ratio is greater than the table value of F, then the difference in sample means is significant.

If the computed value of F −ratio is less than the table value of F, then the difference in sample means is not significant.

$$If \quad F_c < F_T \quad - \quad Difference\ not\ significant$$
$$If \quad F_c > F_T \quad - \quad Difference\ is\ significant$$

10. Define replication.

Solution:

To estimate the magnitude of an effect in an experiment the principle of randomization and replication are applied Randomization by itself is not necessarily sufficient to yield a valid experiment.

The replication or repetition of an experiment or experimental unit is also necessary. Randomization must be invariably accompanied by sufficient replication so as to ensure validity in an experiment.

11. Write the ANOVA table for randomized block design.

Solution:

Source of variation	Sum of squares	Degrees of freedom	Mean sum of square	F-ratio
Between Columns	SSC	$v_1 = C - 1$	$MSC = \frac{SSC}{v_1}$	$F_c = \frac{MSC}{MSC}$
Between Rows	SSR	$v_2 = R - 1$	$MSR = \frac{SSR}{v_2}$	$F_R = \frac{MSR}{MSE}$
Error	SSE	$v_3 = v_1 * v_2$	$MSE = \frac{SSE}{v_3}$	

12. What is the aim of experiment design?

Solution:

The experiment design is test or series of tests in which purposeful changes are made to the input variables of a process or system so that we may observe and identify the reasons for changes that might have occurred in the output response.

13. When do you apply the analysis of variance technique?

Solution:

Suppose we consider three or more samples at a time, in this situation we need another testing hypothesis that all the samples are drawn from the same population, i.e., they have the same means. In this case we use analysis of variance to test the homogeneity of several means.

14. What is meant by a completely randomized design?

Solution:

The term one-way classification of CRD refers to the fact that a single variable factor of interest is controlled and its effect on the elementary units is observed.

15. Discuss the advantages and disadvantages of randomized block design.

Solution:

a. Evaluation and comparison of basic design of configurations.

b. Evaluation of material alternatives.

c. Determination of key product design parameters that affect performance.

Also it uses to improve manufacturing a product, field performance, reliability and lower product cost, etc.

16. Bring out any two advantages between RBD over CRD.

Solution:

i. This design is more efficient than CRD. That is it has less experimental error.

ii. The statistical analysis for this design is simple and rapid.

17. What is Latin square design? Under what conditions can this design are used?

Solution:

The n treatments are then allocated at random to these rows and columns in such a way that every treatment occurs once and only once in each row and in each column. Such a layout is known as $n \times n$ Latin square design.

18. What is the main advantage of LSD over RBD?

Solution:

The main advantage of LSD is that it controls the variances between the rows and columns, whereas RBD controls the effect of one direction (either row or column) and hence the experimental error is reduced to a large extent.

19. Compare RBD and LSD.

Solution:

i. RBD is more efficient / accurate than CRD for most types of experimental work.

ii. RBD is more flexible than CRD.

iii. No restrictions are placed on the number of treatments.

20. Write any two differences between LSD over RBD?

Solution:

i. In LSD, the no. of treatments is equal to the no. of replications whereas there is no such restriction on treatments and replications in RBD.

ii. The main advantage of LSD is that it controls the variances between the rows and columns, whereas RBD controls the effect of one direction (either row or column).

21. Write the ANOVA table for Latin Square design.

Solution:

Source of variation	Sum of squares	Degrees of freedom	Mean sum of square	F-ratio
Between Columns	SSC	$v_1 = n-1$	$MSC = \frac{SSC}{v_1}$	$F_c = \frac{MSC}{MSC}$
Between Rows	SSR	$v_2 = n-1$	$MSR = \frac{SSR}{v_2}$	$F_R = \frac{MSR}{MSE}$
Between treatments	SSK	$v_3 = n-1$	$MSK = \frac{SSK}{v_3}$	$F_k = \frac{MSK}{MSE}$
Error	SSE	$v_4 = (n-1)(n-2)$	$MSE = \frac{SSE}{v_4}$	

22. Give the layout of Latin square design with four treatments.

With four treatments A, B, C and D one typical arrangement of 4×4 LSD is as follows

A	B	D	C
B	A	C	D
D	C	B	A
C	D	A	B

23. **Discuss the advantages and disadvantages of randomized block design.**

Solution:

a. Evaluation and comparison of basic design of configurations.

b. Evaluation of material alternatives.

c. Determination of key product design parameters that affect performance.

Also it uses to improve manufacturing a product, field performance, reliability and lower product cost, etc.

24. State the advantages of a factorial experiment over a simple experiment.

Solution:

Factorial designs are frequently used in experiments involving several factors where it is necessary to the joint effect of the factors on a response.

25. Compare One way classification modal with Two way classification modal.

Solution:

	One way	Two way
1	We cannot test two sets of Hypothesis	Two sets of hypothesis can tested.
2	Data are classified according to one factor	Data are classified according to two different factor.

26. What is meant by Latin square?

Solution:

The n treatments are then allocated at random to these rows and columns in such a way that every treatment occurs once and only once in each row and in each column. Such a layout is known as $n \times n$ Latin square design.

27. Define Mean sum of squares.

The sum of square divided by its degree of freedom given the corresponding variance or the mean sum of squares (MSS).

$E.g., \quad MSC = \frac{SSC}{SSE}$

28. What are the disadvantages of a CRD?

- CRD results in the maximum use of the experimental units since all the experimental material can be used.
- The design is very flexible. Any number of treatments can be used and different treatments can be used unequal number of times without unduly complicating the statistical analysis in most of the cases.

UNIT – III SOLUTIONS OF EQUATIONS AND EIGEN VALUE PROBLEMS

1. What is the order of convergence of Newton- Raphson method if the multiplicity of the root is one.

Solution : The order of convergence of Newton-Raphson method is 2

2. What is the rate of convergence in Newton-Raphson method ?

Solution : The rate of convergence in Newton-Raphson method is 2.

3. What is the criterion for convergence of Newton- Raphson –method ?

Solution :

The Criterion for convergence of Newton- Raphson -method is

$$|f(x)\, f''(x)| < |\, f'(x)|^2 \quad \textit{in the interval considered.}$$

4. Write the iterative formula for Newton- Raphson –method.

Solution:

The Newton- Raphson formula is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

5. In what form is the coefficient matrix transformed into when $A\,X = B$ is solved by Gauss – elimination method?
Solution: Upper triangular matrix.

6. In what form is the coefficient matrix transformed into when $A\,X = B$ is solved by Gauss – Jordan method?
Solution: Diagonal matrix.

7. Explain briefly Gauss – Jordan iteration to solve simultaneous equation?
Solution :
Consider the system of equations $A\,X = B$.
If A is a square matrix the given system reduces to

$$\begin{pmatrix} a_{11} & 0 & \cdots & \cdots & 0 \\ 0 & a_{21} & \cdots & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & 0 & 0 & 0 & a_{nn} \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \cdots \\ b_n \end{bmatrix}$$

This system is reduces to the following n equations.

$$a_{11}x_1 = b_1, \quad a_{22}x_2 = b_2, \dots \dots \; a_{nn}x_n = b_n$$
$$x_1 = \frac{b_1}{a_{11}}, \quad x_2 = \frac{b_2}{a_{22}}, \dots \dots \; x_n = \frac{b_n}{a_{nn}}$$

The method of obtaining the solution of the system of equations by reducing the matrix A to diagonal matrix is known as Gauss – Jordan elimination method.

8. For solving a linear system, compare Gauss - elimination method and Gauss – Jordan method.
Solution:

	Gauss - elimination method	Gauss – Jordan method
1	Coefficient matrix transformed into upper triangular matrix	Coefficient matrix transformed into diagonal matrix
2	Direct method	Direct method
3	We obtain the solution by Backward substitution method	No need of Backward substitution method

9. State the principle used in Gauss – Jordan method.
Solution:
Coefficient matrix transformed into upper triangular matrix

10. Write the sufficient condition for Gauss - Siedal method to converge.
Solution:
The coefficient of matrix should be diagonally dominant.

$$|a_1| > |b_1| + |c_1|, \; |b_2| > |a_2| + |c_2|, \quad |c_3| > |a_3| + |b_3|$$

11. State the sufficient condition for Gauss - Jacobi method to converge.
Solution:
The coefficient of matrix should be diagonally dominant.

$$|a_1| > |b_1| + |c_1|, \; |b_2| > |a_2| + |c_2|, \quad |c_3| > |a_3| + |b_3|$$

12. Give two indirect methods to solve a system of linear equations.

Solution:

(1). Gauss – Jacobi method

(2). Gauss – Siedal method

13. Compare Gauss – Jacobi method and Gauss – Siedal method

Solution:

	Gauss – Jacobi method	Gauss – Siedal method
1	Convergence rate is slow	The rate of convergence of Gauss – Siedal method is roughly twice that of Gauss - Jacobi
2	Indirect method	Indirect method
3	Condition for the convergence is the coefficient matrix is diagonally dominant	Condition for the convergence is the coefficient matrix is diagonally dominant

14. Find the first approximation to the root lying between 0 and 1 of $x^3 + 3x - 1 = 0$ by Newton's method.

Solution: $f(x) = x^3 + 3x - 1 \ and \ f'(x) = 3x^2 + 3$

$$x_1 = x_n - \frac{f(x_0)}{f'(x_0)} \implies 0 - \left(\frac{-1}{3}\right) = 0.3333.$$

15. Find an iteration formula for finding the square root of N by Newton method.

Solution: $f(x) = x^2 - N \ and \ f'(x) = 2x$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\implies x_n - \frac{({x_n}^2 - N)}{2 x_n} = \left(\frac{1}{2}\right)\left(x_n + \frac{N}{x_n}\right), n = 0,1,2,\ldots$$

16. Is the system of equations $3x + 9y - 2z = 10,\ 4x + 2y + 13z = 1,\ 4x - 2y + z = 3$ Diagonally dominant? If not make it diagonally dominant.

Solution:

Let $x + 9y - 2z = 10,\ 4x + 2y + 13z = 1, 4x - 2y + z = 3$

$|a_1| > |b_1| + |c_1|,\ |b_2| > |a_2| + |c_2| \ and \ |c_3| > |a_3| + |b_3|$

Hence the given system is not diagonally dominant.

Hence we rearrange the system as follows $4x - 2y + z = 3$

$x + 9y - 2z = 10,$

$4x + 2y + 13z = 1$

17. Explain power method to find the dominant Eigen value of a square matrix.

Solution:

If v_0 is an initial arbitrary vector, then compute $y_{k+1} = A\, v_k \ and \ v_{k+1} = \frac{Y_{k+1}}{m_{k+1}}$ where m_{k+1} is the numerically largest element of y_{k+1}. Then dominant Eigen value

$\lambda_1 = \lim_{n\to\infty} \frac{[Y_{k+1}]_i}{[v_k]_i}, i = 1,2,\ldots n$

18. Write Newton's formula for finding cube root of N.

Solution:

$$x^3 - N = 0 \quad \Longrightarrow \quad f(x) = x^3 - N \quad and \quad f'(x) = 3\,x^2$$

$$By\ newton's method, we\ have \quad x_{n+1} = x_n - \frac{{x_n}^3 - N}{3\,{x_n}^2}\ , \quad n = 1,2, \ldots$$

19. Write Newton's formula for finding reciprocal of a positive number N.

Solution:

$$N = \frac{1}{x} \quad \Longrightarrow \quad f(x) = \frac{1}{x} - N \quad and \quad f'(x) = \frac{-1}{x^2}$$

$$By\ newton's method \quad x_{n+1} = x_n - \frac{\left(\frac{1}{x_n} - N\right)}{\frac{-1}{{x_n}^2}} = x_n(2 - N\,x_n), \qquad n = 1,2, \ldots$$

UNIT – III

INTERPOLATION,

1. State Lagrange's interpolation formula.

Answer: Let $y = f(x)$ be a function which takes the values $y_0, y_1, \ldots \ldots y_n$ corresponding $to\ x = x_0, x_1, x_2, \ldots \ldots \ldots$.

Then, Lagrange's interpolation formula is

$$y(x) = \frac{(x - x_1)(x - x_2) \ldots (x - x_n)}{(x_0 - x_1)(x_0 - x_2) \ldots (x_0 - x_n)}\ y_0 + \frac{(x - x_0)(x - x_2) \ldots (x - x_n)}{(x_1 - x_0)(x_1 - x_2) \ldots (x_1 - x_n)}\ y_1 + \frac{(x - x_0)(x - x_1) \ldots (x - x_n)}{(x_2 - x_0)(x_2 - x_1) \ldots (x_2 - x_n)}\ y_2 + \cdots \ldots . + \frac{(x - x_0)(x - x_1) \ldots (x - x_{n-1})}{(x_n - x_0)(x_n - x_1) \ldots (x_n - x_{n-1})}\ y_n$$

2. What is the Lagrange's interpolation formula to find y, if three sets of values $(x_0, y_0), (x_1, y_1)$ & (x_2, y_2) are given.

Answer:

$$y(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)}\ y_0 + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)}\ y_1 + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)}\ y_2$$

3. What is the assumption we make when Lagrange's formula is used?

Answer:

Lagrange's interpolation formula can be used whether the vales of x, the independent variable are equally spaced or not whether the difference of y become smaller or not.

4. What advantages has Lagrange's interpolation formula over Newton?

Answer:

The forward and backward interpolation formulae of Newton can be used only when the values of the independent variable x are equally spaced can also be used when the differences of the independent variable y

become smaller ultimately. But Lagrange's interpolation formula can be used whether the values of x, the independent variable are equally spaced or not and whether the difference of y become smaller or not.

5. What is the disadvantage in practice in applying Lagrange's interpolation formula?

Answer:

Through Lagrange's interpolation formula is simple and easy to remember, its application is not speedy. It requires close attention to sign and there is always a chance of committing some error due to a number of positive and negative signs in the numerator and the denominator.

6. What is inverse interpolation?

Answer: Suppose we are given a table of vales of x and y. Direct interpolation is the process of finding the values of y corresponding to a value of x, not present in the table. Inverse interpolation is the process of finding the values of x corresponding to a value not present in the table.

7. State Lagrange's inverse interpolation formula.

Answer:

$$x = \frac{(y-y_1)(y-y_2)\dots(y-y_n)}{(y_0-y_1)(y_0-y_2)\dots(y_0-y_n)}\,x_0 + \frac{(y-y_0)(y-y_2)\dots(y-y_n)}{(y_1-y_0)(y_1-y_2)\dots(y_1-y_n)}\,x_1 + \frac{(y-y_0)(y-y_1)\dots(y-y_n)}{(y_2-y_0)(y_2-y_1)\dots(y_2-y_n)}\,x_2$$
$$+\cdots\dots+\frac{(y-y_0)(y-y_1)\dots(y-y_{n-1})}{(y_n-y_0)(y_n-y_1)\dots(y_n-y_{n-1})}\,x_n$$

8. Define 'Divided Differences'.

Answer: Let the function $y=f(x)$ take the values $f(x_0), f(x_1), \dots f(x_n)$ corresponding to the values $x_0, x_1, x_2, \dots\dots. x_n$ of the argument x where $x_1 - x_0,\ x_2 - x_1, \dots\dots x_n - x_{n-1}$ $need$ not necessarily be equal.

The first divided difference of $f(x)$ for the arguments x_0, x_1 is

$$f(x_0,x_1) = \frac{f(x_1)-f(x_0)}{x_1-x_0} \qquad f(x_1,x_2) = \frac{f(x_2)-f(x_1)}{x_2-x_1}$$

9. Form the divided for the following data

x :	2	5	10
y :	5	29	109

Solution: The divided difference table is as follows

x :	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$
2	5	$\frac{29-5}{5-2}=8$	$\frac{16-8}{10-2}=1$
5	29	$\frac{109-29}{10-5}=16$	
10	109		

10. Form the divided for the following data

x :	5	15	22
y :	7	3629	160

Solution: The divided difference table is as follows

x :	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$
5	7		
		$\frac{36-7}{15-5} = 2.9$	
15	36		$\frac{17.7-2.9}{22-5} = 2.114$
		$\frac{160-36}{22-15} = 17.7$	
22	160		

11. Give the Newton's divided difference formula.

Solution :

$$f(x) = f(x_0) + (x - x_0)f(x_0, x_1) + (x - x_0)\,(x - x_1)f(x_0, x_1, x_2) + (x - x_0)\,(x - x_1)\,(x - x_2)f(x_0, x_1, x_2, x_3) + \cdots \dots \dots .. + (x - x_0)(x - x_1) \dots (x - x_n)\, f(x_0, x_1, \dots x_{n-1})$$

12. State any properties of divided differences.

Solution:

(1). The divide difference are symmetrical in all their arguments. That is the value of any difference is independent of the order of the arguments.

(2). The divided difference of the sum or difference of two functions is equal to the sum or difference of the corresponding separate divided differences.

13. When Newton's Backward interpolation formula used.

Solution:

The formula is used mainly to interpolate the values of y near the end of a set of tabular values and also for extrapolating the values of y a short distance ached (to the right) of y_n.

14. Newton's Backward interpolation formula used only for ……………………..?

Solution:

Equidistant intervals (or) Equal intervals.

15. Say True or False.

Newton's interpolation formulae are not suited to estimate the value of a function near the middle of a table.

Answer: True.

16. Say True or False.

Newton's forward and Newton's backward interpolation formulae are applicable for interpolation near the beginning and end respectively of tabulated values.

Answer: The statement is true.

17. When will we use Newton's forward interpolation formula?

Solution:

The formula is used to interpolate the values of y near the beginning of the table value and also for extrapolating the values of y short distance (to the left) ahead of y_0.

UNIT- IV INTERPOLATION, NUMERICAL DIFFERENTIATION & INTEGRATION

1. State Newton's forward Difference formula to find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = x_0$.

Answer:

$$\left(\frac{dy}{dx}\right) = \frac{1}{h}\left\{\Delta y_0 - \frac{\Delta^2 y_0}{2} + \frac{\Delta^3 y_0}{3} - \cdots\right\} \quad and$$

$$\left(\frac{d^2y}{dx^2}\right) = \frac{1}{h^2}\left\{\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\ \Delta^4 y_0 - \cdots\right\}$$

2. Find the parabola of the form $y = a\,x^2 + bx + c$ passing through the points $(0,0), (1,1)\ \&\ (2,20)$.

Answer:

Let us known $f(x_0) = y_0$.

Here $y_0 = 0,\ \Delta y_0 = 1,\ \ \Delta y_1 = 19,\ \Delta^2 y_0 = 18,\ p = x$

$$f(x) = 0 + x(1) + \frac{x(x-1)}{2}(18) \Longrightarrow y = 9\,x^2 - 8x$$

3. Write the formula to compute $\frac{dy}{dx}$ at $x = x_0 + ph$ for the given data

$(x_i, y_i), i = 0,1,2, \dots n.$

Answer:

$$\left(\frac{dy}{dx}\right) = \frac{1}{h}\left\{\Delta y_0 - \frac{2p-1}{2}\ \Delta^2 y_0 + \frac{3p^2-6p+2}{6}\ \Delta^3 y_0 + \frac{2p^3-9p^2+11p-3}{12}\ \Delta^4 y_0 - \cdots\right\}$$

$where\ \ p = \frac{x-x_0}{h.}$

4. Write the formula to compute $\frac{d^2y}{dx^2}$ at $x = x_0 + ph$ for the given data $(x_i, y_i), i = 0, 1, 2, \dots n.$

Answer:

$$\left(\frac{d^2y}{dx^2}\right) = \frac{1}{h^2}\left\{\Delta^2 y_0 + (p-1)\,\Delta^2 y_0 + \frac{6p^2 - 18\,p + 11}{12}\ \Delta^4 y_0 + \cdots\right\}$$

$where\ \ p = \frac{x-x_0}{h.}$

5. State Newton's Backward interpolation formula to find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = x_n$.

Answer:

$$\left(\frac{dy}{dx}\right)_{x=x_n} = \frac{1}{h}\left\{\nabla y_n + \frac{\nabla^2 y_n}{2} + \frac{\nabla^3 y_n}{3} + \cdots\right\} \quad and$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_n} = \frac{1}{h^2}\left\{\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\ \nabla^4 y_0 - \cdots\right\}$$

6. Write the formula to compute $\frac{dy}{dx}$ at $x = x_n + ph$ for the given data $(x_i, y_i), i = 0, 1, 2, \dots n.$

Answer:

$$\left(\frac{dy}{dx}\right) = \frac{1}{h}\left\{\nabla y_n - \frac{2p+1}{2}\ \nabla^2 y_n + \frac{3+6p+2}{6}\ \nabla^3 y_n + \frac{2p^3+9p^2+11p+3}{12}\ \nabla^4 y_0 - \cdots\right\}$$

$where\ \ p = \frac{x-x_n}{h.}$

7. Write the formula to compute $\frac{d^2y}{dx^2}$ at $x = x_n + ph$ for the given data $(x_i, y_i), i = 0, 1, 2, \dots n$.

Answer:

$$\left(\frac{d^2y}{dx^2}\right) = \frac{1}{h^2}\left\{\nabla^2 y_n + (p+1)\,\nabla^3 y_n + \frac{6p^2+18\,p+11}{12}\ \nabla^4 n + \cdots\right\}$$

$where\ \ p = \frac{x-x_n}{h.}$

8. Find $\frac{dy}{dx}$ at $x = 2$ from the following data.

$x:$	2	3	4
$y:$	26	58	112

Answer: $\Delta y_0 = 32, \Delta y_1 = 54\,, \Delta^2 y_0 = 22$

$$\frac{dy}{dx} = 32 - \frac{1}{2}\,(22) = 21.$$

10. Find $\frac{dy}{dx}$ at $x = 6$ from the following data.

$x:$	2	4	6
$y:$	3	11	27

Answer: $\nabla y_n = 16,\ \nabla y_{n-1} = 8,\ \nabla^2 y_n = 16 - 8 = 8$

$$\left(\frac{d^2y}{dx^2}\right)_{x=6} = \frac{1}{2}\left(16 + \frac{8}{2}\right) = 10.$$

11. A curve passing through the points $(1, 0), (2, 1)$ and $(4, 5)$. Find the slope of the curve at $x = 3$.

Answer:

$$f(1{,}2) = 1, \qquad f(2{,}4) = 2, \qquad f(1{,}2{,}4) = \frac{1}{3}.$$

$$f(x) = f(x_0) + (x - x_0)f(x_0, x_1) + (x - x_0)(x - x_1)f(x_0, x_1, x_2)$$

$$f(x) = 0 + (x-1)(1) + (x-1)(\,x-2)\frac{1}{3}\ =\ x - 1 + \frac{1}{3}\,(x^2 - 3x + 2)$$

$$f'^{(x)} = 1 + \frac{2x}{3} - 1 = \frac{2x}{3}$$

Slope at $x = 3$ is $\frac{2(3)}{3} = 2.$

12. State Trapezoidal rule with the error order.

Answer: For the given data (x_i, y_i) where $x_i = x_0 + ih,\ \ i = 0{,}1{,}2 \dots .n$

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{3}\,[(y_0 + y_n) + 2(y_1 + y_3 + \cdots + y_{n-1})] \qquad and$$

$$Error\ is\ of\ order\ h^2\ .$$

13. State Simpson's $\frac{1}{3}$ $\;$ *and* $\frac{3}{8}$ rule.

Answer:

If (x_i, y_i) $\;i = 0,1,2, \ldots n$ $\;$ where $x_i = x_0 + ih,\; then$

Simpson's $\frac{1}{3}$ rule :

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{3}\,[(y_0 + y_n) + 4(y_1 + y_3 + y_5 + \cdots +) + 2(y_2 + y_4 + y_6 + \cdots)]$$

Simpson's $\frac{3}{8}$ rule :

$$\int_{x_0}^{x_n} y\,dx = \frac{3h}{8}\,[(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5 + \cdots +) + 2(y_3 + y_6 + y_9 \ldots)]$$

14. State the order of error in Simpson's $\frac{1}{3}$ rule.

Answer:

Error in Simpson's $\frac{1}{3}$ rule is of order h^4.

15. Using Simpson's rule, find $\int_0^4 e^x\,dx$ given $e^0 = 1, e^1 = 2.72,\; e^2 = 7.39,\; e^3 = 20.09\;\&\; e^4 = 54.6.$

Answer:

$$\int_0^4 e^x\,dx = \frac{1}{3}\,[(1 + 54.6) + 4(2.72 + 20.09) + 2(7.39)] = 53.873.$$

16. A curve passes through $(2, 8),\; (3, 27),\; (4, 64)\;\&\; (5, 125)$, find the area of the curve between x- axis and the line $x = 2$ *and* $x = 5$, by Trapezoidal rule.

Answer:

$$\int_2^5 y\,dx = \frac{1}{2}\,[(8 + 125) + 2(27 + 64)] = 157.5\; sq.units.$$

18. Find $\int_{-2}^{+2} x^4\,dx$ by Simpson's rule, taking h=1.

Answer:

$$\begin{array}{llllll} x: & -2 & -1 & 0 & 1 & 2 \\ y: & 16 & 1 & 0 & 1 & 16 \end{array}$$

$$\therefore \int_{-2}^{+2} x^4\,dx = \frac{1}{3}\,[(16 + 16) + 4(2)] = 13.3\;\; sq.units.$$

19. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by Trapezoidal rule with $h = 0.5$.

Answer :

$$\int_0^1 \frac{dx}{1 + x^2} = \frac{0.5}{2}\,[1.5 + 2(0.8)] = 0.775.$$

20. Use Simpson's $\frac{1}{3}$ rule with $h = 0.5$ to evaluate $\int_0^1 \frac{dx}{1+x}$.

Answer:

$$\int_0^1 \frac{dx}{1+x} = \frac{1}{6}\left[1 + \frac{4}{1.5} + \frac{1}{2}\right] = 0.6944.$$

21. Evaluate $\int_{-1}^{+1} |x| dx$ with two subintrevals by Simpson's $\frac{1}{3}$ rule and by Trapezoidal rule.

Answer:

By Simpson's $\frac{1}{3}$ rule $I = \frac{1}{3}[1 + 0 + 1] = \frac{2}{3}$

By Trapezoidal rule $I = \frac{1}{2}[1 + 1] = 1$

22. State the errors & order for Simpson's rule and Trapezoidal rules.

Solution:

Rule	Degree of $y(x)$	No.of Intrevals	Error	Order
Trapezoidal rule	One	Any	$\lvert E\rvert < \frac{(b-a)h^2}{12} M$	h^2
Simpson's $1/3$rule	Two	Even	$\lvert E\rvert < \frac{(b-a)h^4}{180} M$	h^4
Simpson's $3/8$ rule	Three	Multiple of 3	$\lvert E\rvert = \frac{3}{8} h^5$	-

UNIT-V

INITIAL VALUE PROBLES FOR ORDINARY DIFFERENTIAL EQUATIONS

01. Write down the order Taylor's algorithim.

Answer :

Let $y' = f(x, y)$ $with$ $y(x_0) = y_0$

Then the Taylor algorithim is given by

$$y(x_1) = y_0 + \frac{h}{1!} y_0' + \frac{h^2}{2!} y_0'' + \frac{h^3}{3!} y_0''' + \frac{h^4}{4!} y_0^{iv} + \cdots$$

$$where \quad x_1 = x_0 + h \quad and \quad {y_0}^{(r)} = \frac{d^r y}{dx^r} \quad at \quad (x_0, y_0).$$

02. What are the merits and demerits of the Taylor series method of solution?

Answer :

It is a powerful single step method.

It is the best method if the expression for higher order derivtives are simpler.

The major demerit of this method is the evaluation of higher order derivatives become tedious for complicated algebric expressions.

03. Given $\boldsymbol{y' = x + y,\ y(0) = 1}$. Find $\boldsymbol{y(0.1)}$ By Taylor's method.

Answer :

$$y' = x + y\,;\ \ y'' = 1 + y';\ \ y''' = y''\ \ldots\ldots$$

$$x_0 = 0, \qquad y_0 = 0.\ Then\ \ y(0.1) = y_1 = y_0 + \frac{h}{1!}\,y_0' + \frac{h^2}{2!}\,y_0'' + \cdots$$

$$when\ \ h = 0.1. \qquad \therefore\ \ y(0.1) = 1 + 0.1 + \frac{0.01}{2}\,(2) + \frac{0.001}{6}\,6\,(2)$$

$$\Rightarrow\ \ y(0.1) = 1.1103$$

04. Find $\boldsymbol{y(0.1)\ by}$ Euler's method, given that $\boldsymbol{y' = 1 - y,\ y(0) = 0}$.

Answer :

$$y_1 = y_0 + h\,f(x_0, y_0) = 0.01[1 - 0] = 0.1 \quad \Rightarrow\ \ y(0.1) = 0.1$$

05. Using Euler's method compute for $\boldsymbol{x = 0.1}$ & $0.2\ \ with\ \ h = 0.1\ \ given$

$y' = y - \frac{2x}{y}\,, y(0) = 1.$

Answer :

$$y_1 = 1 + 0.1[1 - 0] = 1.10 \quad \Rightarrow\ \ y(0.1) = 1.10$$

$$y_2 = 1.1 + 0.1\left[1.1 - \frac{0.2}{1.1}\right] = 1.19 \quad \Rightarrow\ \ y(0.2) = 1.19$$

06. Find $\boldsymbol{y(0.1)\ by}$ Euler's method, given that $\boldsymbol{y' = x + y,\ y(0) = 1}$.

Answer :

$$y_1 = 1 + 0.1[0 + 1] = 1.10 \quad \Rightarrow\ \ y(0.1) = 1.10$$

07. Given $\boldsymbol{y' + y = 0\ \ and\ \ y(0) = 1}$. Find $\boldsymbol{y(0.01)\ and\ y(0.02)}$ by Euler's method

Answer :

$$y_1 = y_0 + h\,f(x_0, y_0) = 0.01[1 - 0] = 0.1 \quad \Rightarrow\ \ y(0.1) = 0.1$$

08. Using Euler's method compute for $\boldsymbol{x = 0.1}$ & $0.2\ \ with\ \ h = 0.1\ \ given$

$y' = y - \frac{2x}{y}\,, y(0) = 1.$

Answer :

$$y_1 = 1 + 0.01[-1] = 0.09 \quad \Rightarrow\ \ y(0.01) = 0.09$$

$$y_1 = 0.99 + 0.01[-0.99] = 0.9801 \quad \Rightarrow\ \ y(0.02) = 0.9801$$

09. State the algorithim for modified Euler's method, to solve $\boldsymbol{\frac{dy}{dx} = f(x, y),\ y(x_0) = y_0}$.

Answer :

$$y_{n+1}^{(1)} = y_n + h\,f(x_n, y_n)$$

$$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f\left(x_{n+1}, y_{n+1}^{(1)}\right)\right]$$

$$where\ \ n = 0,1,2, \ldots \quad and \quad x_{n+1} = x_n + h$$

10. State Rung-Kutta fourth order formulae for solving $\boldsymbol{\frac{dy}{dx} = f(x, y),\ y(x_0) = y_0}$.

Answer :

$$y_1 = y(x_0 + h) = y_0 + \frac{1}{6}\,[k_1 + 2k_2 + 2k_3 + k_4] \quad where$$

$$k_1 = h\,f(x_0, y_0)$$

$$k_2 = h\,f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$k_3 = h\,f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$

$$k_4 = h\,f(x_0 + h, y_0 + k_3)$$

11. What are th distinguish property for Rung-Kutta methods?

Answer :

(1). These methods do not require the higher order derivatives and requires only the function values at different points.

(2). To evaluate y_{n+1}, we need only y_n but not previous of y's.

(3). The solution by these methods agree with Taylor series solution upto the terms of h^r

Where r is the order of Runge-Kutta method.

12. Which is the better Taylor series method or Runge – Kutta method? Why?

Answer :

Runge-Kutta method is better since higher order derivatives of y are not required. Taylor's series method involves use of higher oder derivatives which may be difficult in case of complicated algebric functions.

13. State the order of error in R-K method of fourth order.

Answer :

Error of fourth order method is $O(h^2)$ where h is the interval of differencing.

14. State Milne's Predictor and Corrector formula.

Answer :

Predictor : $y_{n+1}^{p} = y_{n-3} + \frac{4h}{3}\,[2\,y'_{n-2} - y'_{n-1} + 2\,y'_n]$

Corrector : $y_{n+1}^{c} = y_{n-1} + \frac{h}{3}\,[\,y'_{n-2} + 4y'_{n-1} + y'_{n+1}]$

15. State Adam's Predictor and Corrector formula.

Answer:

Predictor : $y_{n+1,P} = y_n + \frac{h}{24}[55\,y'_n - 59y'_{n-1} + 37\,y'_{n-2} - 9y'_{n-3}]$

Corrector : $y_{n+1,C} = y_n + \frac{h}{24}\,[9\,y'_{n+1} + 19y'_n - 5\,y'_{n-1} + y'_{n-2}]$

16. Write the predictor error and corrector error in Milne's method.

Answer:

Predictor error $= \frac{14}{45}\,h^5 f^{iv}(\mathcal{E})$

Corrector error $= -\frac{h^5}{90}\,y^{iv}(\mathcal{E})$

17. Distinguish Single – step and Multi – step methods.

Answer:

Single – step methods: To find y_{n+1} , the information at y_n is enough.

Multi – step methods: To find y_{n+1} , the past four values $y_{n-3}, y_{n-2}, y_{n-1}$ *and* y_n are needed.

18. State the finite approximations for y' & y'' with error terms.

Answer:

$$y_i = y(x_i) \;\; and \;\; x_{i+1} = x_i + h, \qquad i = 0,1,2,\dots n$$

$$Then \quad y_i' = \frac{y_{i+1} - y_{i-1}}{2h}, \qquad Error = O(h^2)$$

$$y_i'' = \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2}, \qquad Error = O(h^2)$$

NUMERICAL METHODS FORMULAS

Newton's Backward Difference Formula

$$y(x_n + vh) = y(x) = y_n + \frac{v}{1!}\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 n + \frac{v(v+1)(v+2)(v+3)}{4!}\nabla^4 y_n + \cdots$$

$$Where \quad v = \frac{x - x_n}{h}$$

Newton's Forward Difference Formula

$$y(x_0 + ph) = y(x) = y_0 + \frac{p}{1!}\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0 + \cdots$$

$$Where \; p = \frac{x - x_0}{h}$$

The First, Second & Third Derivatives at the initial position x is

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 + \frac{2p-1}{2}\Delta^2 y_0 + \frac{3p^2 - 6p + 2}{6}\Delta^3 y_0 + \frac{4p^3 - 18p^2 + 22p - 6}{24}\Delta^4 y_0 + \cdots\right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_0 + (p-1)\Delta^3 y_0 + \frac{6p^2 - 18p + 11}{12}\Delta^4 y_0 + \cdots\right]$$

$$\frac{d^3y}{dx^3} = \frac{1}{h^3}\left[\Delta^3 y_0 + \frac{12p - 18}{12}\Delta^4 y_0 + \cdots\right]$$

The First, Second & Third Derivatives at the starting value $x = x_0$ is (Initial Point)

Since $at \;\; x = x_0, \;\; p = 0, \;\; we \; have$

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \cdots\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 + \cdots\right]$$

$$\left(\frac{d^3y}{dx^3}\right)_{x=x_0} = \frac{1}{h^3}\left[\Delta^3 y_0 - \frac{3}{2}\,\Delta^4 y_0 + \cdots\right]$$

Newton's Backward Difference Formula

$$y(x_n + vh) = y(x) = y_n + \frac{v}{1!}\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 n + \frac{v(v+1)(v+2)(v+3)}{4!}\nabla^4 y_n + \cdots$$

$$Where \quad v = \frac{x - x_n}{h}$$

The First, Second & Third Derivatives at the ending value x is

$$\frac{dy}{dx} = \frac{1}{h}\left[\nabla y_n + \frac{2v+1}{2}\nabla^2 y_n + \frac{3v^2+6v+2}{6}\nabla^3 y_n + \frac{4v^3+18v^2+22v+6}{24}\nabla^4 y_n + \cdots\right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\nabla^2 y_n + (v+1)\nabla^3 y_n + \frac{6v^2+18v+11}{12}\nabla^4 y_n + \cdots\right]$$

$$\frac{d^3y}{dx^3} = \frac{1}{h^3}\left[\nabla^3 y_n + \frac{12v+18}{12}\nabla^4 y_n + \cdots\right]$$

The First, Second & Third Derivatives at the ending value $x = x_n$ is (Ending Point)

Since $at\ x = x_n, \quad v = 0, \ we\ have$

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\,\nabla^3 y_n + \frac{1}{4}\nabla^4 y_n + \cdots\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \cdots\right]$$

$$\left(\frac{d^3y}{dx^3}\right)_{x=x_0} = \frac{1}{h^3}\left[\nabla^3 y_n + \frac{3}{2}\,\nabla^4 y_n + \cdots\right]$$

The First, Second & Third Derivatives at the starting value $x = x_0$ is (Initial Point)

Since $at\ x = x_0, \quad p = 0, \ we\ have$

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\frac{1}{2}(\Delta y_0 + \Delta y_{-1}) - \frac{1}{12}(\Delta^3 y_{-1} + \Delta^3 y_{-2}) + \frac{1}{60}\,(\Delta^5 y_{-2} + \Delta^5 y_{-3}) + ..\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2}\left[\Delta^2 y_{-1} - \frac{1}{12}\Delta^4 y_{-2} + \cdots\right]$$

$$\left(\frac{d^3y}{dx^3}\right)_{x=x_0} = \frac{1}{h^3}\left[\frac{1}{2}(\Delta^3 y_{-1} + \ \Delta^3 y_{-2}) + \cdots\right]$$

Trapezoidal rule

$$I = \int_a^b f(x)\,dx = \frac{h}{2}\,[sum\ of\ first\ \&\ last\ ordinates + 2(sum\ of\ remailning\ ordinates)]$$

$$I = \int_a^b f(x)\,dx = \frac{h}{2}\,[(y_0 + y_n) + 2(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + \cdots)]$$

The Error Order of Trapezoidal rule is h^2

Simpson's rule 1/3 Rule

$$I = \int_a^b f(x)\,dx = \frac{h}{2}\,[(sum\ of\ first\ \&\ last\ ordinates) + 4(sum\ of\ remailning\ odd\ ordinates)$$
$$+ 2(sum\ of\ remailning\ even\ ordinates)]$$

$$I = \int_a^b f(x)\,dx = \frac{h}{2}\,[(y_0 + y_n) + 4(y_1 + y_3 + y_5 + y_7 + \cdots) + 2(y_2 + y_4 + y_6 + y_8 + \cdots)]$$

The Error Order of Simpson's rule is h^4

Simpson's rule 3/8 Rule

$$I = \int_a^b f(x)\,dx = \frac{3h}{8}\,[(y_0 + y_n) + 4(y_1 + y_2 + y_4 + y_5 + \cdots + y_{n-1}) + 2(y_3 + y_6 + \cdots + y_{n-3})]$$

The Taylor's Series expansion of $\boldsymbol{y(x)}$ at $\boldsymbol{x = x_0}$ is given by

$$y' = \frac{dy}{dx} = f(x,y), \quad y(x_0) = y_0$$

$$y(x) = y_0 + \frac{(x - x_0)}{1\,!}\,y_0' + \frac{(x - x_0)^2}{2\,!}\,y_0'' + \frac{(x - x_0)^3}{3\,!}\,y_0''' + \frac{(x - x_0)^4}{4\,!}\,y_0^{iv} + \cdots$$

Euler's Method

$$y_{n+1}(x_n + h) = y_n + h\,f(x_n, y_n) \quad , \qquad n = 0,1,2 \ldots.$$

Modified Euler's Method

$$y_{n+1}(x_n + h) = y_n + h\,f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}\,f(x_n, y_n)\right) \quad , \qquad n = 0,1,2 \ldots.$$

Improved Euler's Method

$$y_{n+1}(x_n+h) = y_n + \frac{h}{2}\,[f(x_n, y_n) + f\{x_n + h, y_n + h\, f(x_n, y_n)\}] \quad , \qquad n = 0,1,2 \ldots.$$

Fourth Order Runge – Kutta Method

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = hf(x_n + h, y_n + k_3)$$

$$\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$y(x_n + h) = y(x_n) + \Delta y \quad , \qquad n = 0,1,2 \ldots.$$

Milne's Predictor & Corrector Methods

$$y' = \frac{dy}{dx} = f(x, y), \quad y(x_0) = y_0$$

$$y_{n+1,P} = y_{n-3} + \frac{4h}{3}[2\,y'_{n-2} - y'_{n-1} + 2y'_n]$$

$$y_{n+1,C} = y_{n-1} + \frac{h}{3}[\,y'_{n-1} + 4y'_n + y'_{n+1}]$$

Adam's Bashforth (Adam's) Predictor & Corrector Methods

$$y' = \frac{dy}{dx} = f(x, y), \quad y(x_0) = y_0$$

$$y_{n+1,P} = y_n + \frac{h}{24}[55\,y'_n - 59\,y'_{n-1} + 37\,y'_{n-2} - 9\,y'_{n-3}]$$

$$y_{n+1,C} = y_n + \frac{h}{24}[9\,y'_{n+1} + 19\,y'_n - 5\,y'_{n-1} + \,y'_{n-2}]$$

Formula for ordinary differential equation of order two $\boldsymbol{ay'' + by' + c = d}$

$$y'' = \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2}$$

$$y' = \frac{y_{i+1} - y_{i-1}}{2h}$$