STATISTICS AND NUMERICAL METHODS

Question IV November / December 2011

Part-A

1. The heights of college students in Chennai are normally distributed with standard deviation 6 cm and sample of 100 students had their mean height 158 cm. test the Hypothesis that the mean height of college students in Chennai is 160 cm at 1% level of significance.

Solution:

Given that $n = 100, \quad \bar{x} = 158, \quad s = 6 \quad and \quad \mu = 160$

Null Hypothesis: $\boldsymbol{H_0: \mu = 160}$ i.e., there is no difference between sample mean and hypothetical population mean.

Alternative Hypothesis: $\boldsymbol{H_1: \mu \neq 160}$

The test statistic is given by

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{158 - 160}{\frac{6}{\sqrt{100}}} = -3.33$$

$$\therefore \quad z = 3.33 \qquad [Calculated\ value]$$

At 1% significance level the tabulated value for $z_\propto$ is 2.58.

$$|Calculated\ value\ | \leq Tabulated\ value \quad then \quad Accept\ H_0$$

$$But \quad |3.33| > 2.58 \quad So\ we\ reject\ H_0.$$

2. A coin is tossed 400 times ant it turns up head 216 times. Discuss whether the coin may be unbiased one at 5% level of significance.

Solution:

Given $n = 400, \quad P = Prob\ of\ getting\ head\ in\ a\ toss = \frac{1}{2}$

X =No.of success= 216

H_0: The coin is unbiased. H_1: The coin is biased.

$$z = \frac{\bar{x} - np}{\sqrt{nPQ}} = \frac{216 - 400\left(\frac{1}{2}\right)}{\sqrt{400\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}} = 1.6$$

$$\therefore \quad z = 1.6 \qquad [Calculated\ value]$$

At 5% significance level the tabulated value for $z_\propto$ is 1.96.

$$|Calculated\ value\ | \leq Tabulated\ value \quad then \quad Accept\ H_0$$

$But \quad |1.6| > 1.96 \quad So\ we\ accept\ H_0$. The coin is unbiased.

3. Define Mean sum of squares.

The sum of square divided by its degree of freedom given the corresponding variance or the mean sum of squares (MSS).

$E.g., \quad MSC = \frac{SSC}{SSE}$

4. What are the disadvantages of a CRD?

- CRD results in the maximum use of the experimental units since all the experimental material can be used.
- The design is very flexible. Any number of treatments can be used and different treatments can be used unequal number of times without unduly complicating the statistical analysis in most of the cases.

5. Find an iterative formula to find $\sqrt{N}$ where N is a positive number and hence find $\sqrt{5}$.

Solution: $$f(x) = x^2 - N \;\; and \;\; f'(x) = 2\,x$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\Rightarrow \; x_n - \frac{(x_n{}^2 - N)}{2\;x_n} = \left(\frac{1}{2}\right)\left(x_n + \frac{N}{x_n}\right), n = 0,1,2,\ldots$$

To find $\sqrt{5}$: $x_{n+1} = \left(\frac{1}{2}\right)\left(x_n + \frac{5}{x_n}\right)$, chose $x_0 = 2$.

$$x_1 = \left(\frac{1}{2}\right)\left(x_0 + \frac{5}{x_0}\right) = \frac{1}{2}\left(2 + \frac{5}{2}\right) = 2.25$$

$$x_2 = \left(\frac{1}{2}\right)\left(x_1 + \frac{5}{x_1}\right) = \frac{1}{2}\left(2.25 + \frac{5}{2.25}\right) = 2.236$$

$$x_2 = 2.236$$

The answer is 2.236.

6. Solve by Gauss Jordan method $\begin{pmatrix} \mathbf{1} & \mathbf{2} & \mathbf{1} \\ \mathbf{0} & \mathbf{-1} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{-8} \end{pmatrix}\begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \end{pmatrix} = \begin{pmatrix} \mathbf{3} \\ \mathbf{4} \\ \mathbf{-24} \end{pmatrix}$.

Solution:

$$[A, I] \sim \left[\begin{array}{ccc:c} -8 & -16 & 0 & \mathbf{0} \\ 0 & 8 & 0 & \mathbf{-8} \\ 0 & 0 & -8 & \mathbf{-24} \end{array}\right] \quad \begin{array}{l} R_1 \Leftrightarrow -8R_1 - R_3 \\ R_2 \Leftrightarrow -8\,R_2 - R_3 \end{array}$$

$$[A, I] \sim \left[\begin{array}{ccc:c} -8 & 0 & 0 & \mathbf{-16} \\ 0 & 8 & 0 & \mathbf{-8} \\ 0 & 0 & -8 & \mathbf{-24} \end{array}\right] \quad R_1 \Leftrightarrow R_1 + 2R_2$$

$\boldsymbol{x = 2}, \;\; \boldsymbol{y = -1}, \;\; \boldsymbol{z = 3}$

7. Find the parabola of the form $\boldsymbol{y = ax^2 + bx + c}$ passing through the points $(\mathbf{0}, \; \mathbf{0}), \; (\mathbf{1}, \; \mathbf{1}) \; \boldsymbol{and} \; (\mathbf{2}, \; \mathbf{20})$.

Solution:

x :	$0 \; x_0$	$1 \; x_1$	$2 \; x_2$
$f(x)$:	$0 \; y_0$	$1 \; y_1$	$20 \; y_3$

$$y = f(x) = \frac{(x-1)(x-2)}{(0-1)(0-2)}(0) + \frac{(x-0)(x-2)}{(1-0)(1-2)}(1) \; + \frac{(x-0)(x-1)}{(2-0)(2-1)}(20)$$

$$f(x) = 0 + (x^2 - 2x)(-1) + 10x(x-1)$$

$$f(x) = 9x^2 - 8x.$$

8. Show that $\Delta^3_{bcd}\left(\frac{1}{a}\right) = -\left(\frac{1}{abcd}\right)$.

Solution:

$$Let \quad f(x) = \frac{1}{x}; \qquad f(a) = \frac{1}{a}$$

$$f(a,b) = \Delta_y(1/a) = \frac{(1/b) - (1/a)}{b-a} = -\frac{1}{ab}$$

$$f(a,b,c) = \Delta^2{}_y(1/a) = \frac{f(b,c) - f(a,b)}{c-a} = \frac{(-1/bc) + (1/ab)}{c-a} = \frac{1}{abc}$$

$$f(a,b,c,d) = \Delta^3{}_y(1/a) = \frac{f(b,c,d) - f(a,b,c)}{d-a} = \frac{(1/bcd) - (1/abc)}{d-a} = \frac{-1}{abcd}$$

9. Using Taylor series method, find $y(1.1)$ correct to four decimal places given $y' = xy^{\frac{1}{3}}$ and $y(1) = 1$.

Solution:

$$y' = xy^{\frac{1}{3}} \qquad\qquad y_0' = 1$$

$$y'' = \frac{1}{3}x\,y^{-\frac{2}{3}} + y^{\frac{1}{3}} \qquad\qquad y_0'' = \frac{4}{3}$$

$$y(x) = y_0 + \frac{(x-x_0)}{1\,!}\,y_0' + \frac{(x-x_0)^2}{2\,!}\,y_0'' +$$

$$y(1.1) = 1 + \frac{(1.1-1)}{1\,!}(1) + \frac{(1.1-1)^2}{2\,!}\left(\frac{4}{3}\right) = 1.106$$

10. Write down the finite difference approximation for the following second order ODE with

$h = \frac{1}{n},\ y'' = y + x,\ y(o) = y(1) = 0.$

Solution:

$$\frac{y_{i+1} + y_{i-1} - 2y_i}{h^2} - y_i = x_i \qquad i.e., \quad n^2 y_{i-1} - (2n^2+1)y_i + n^2 y_{i+1} = x_i$$

Part-B

11. (a) (i). Two sample polls of votes for two candidates A and B for a public office are taken from among residents of rural areas. The results are given below. Examine whether the nature of the area is related to voting performance in this election.

Area/Votes for	A	B	Total
Rural	620	380	1000
Urban	550	450	1000
Total	1170	830	2000

Solution:

H_0: The nature of the area and voting performance are independent.

The test statistic is given by

$$\chi^2 = \sum_{i=1}^{r}\sum_{j=1}^{s}\frac{\left(O_{ij} - E_{ij}\right)^2}{E_{ij}} \qquad \chi^2\ distribution\ with\ \ n = (r-1)(s-1)\ \ d.o.f$$

$$E_{ij} = \frac{R_i C_j}{N} \;;\quad i = 1,2,\ldots r \;\; and \;\; j = 1,2,\ldots.s$$

The expected frequencies are

$$E(620) = \frac{1170 * 1000}{2000} = 585\;; \qquad E(380) = \frac{830 * 1000}{2000} = 415$$

$$E(550) = 585; \qquad E(450) = 415$$

O_{ij}	E_{ij}	$(O_{ij} - E_{ij})$	$\frac{(O_{ij} - E_{ij})^2}{E_{ij}}$
620	585	35	2.094017
380	415	-35	2.951807
550	585	-35	2.094017
450	415	35	2.951807

$$\boldsymbol{\chi^2 = 10.08}$$

Table value of $\chi^2_{0.05}$ with $n = (r-1)(s-1) = (2-1)(2-1) = 1$ d.o.f is 3.84.

Conclusion:

Since $\chi^2 > \chi^2_{0.01}$, we reject null hypothesis. That is some relationship between area and vote performance.

11. (a). (ii). Fit a Poisson distribution to the following data and test the goodness of fit.

X:	0	1	2	3	4	5	6
Frequencies:	275	72	30	7	5	2	1

Solution:

The Poisson distribution function is given by

$$P(X = x) = \frac{e^{-\lambda}\,\lambda^x}{x!}, \qquad x = 0,1,1,2\ldots$$

x:	0	1	2	3	4	5	6	$N = \sum f = 392$
$f(x)$;	275	72	30	7	5	2	1	
$f(x) * x$	0	72	60	21	20	10	6	$\sum f * x = 189$

$$\lambda = \frac{\sum f * x}{\sum f} = \frac{392}{189} = 2.07$$

The expected frequencies are given by

$$E_i = P(x) = N * \frac{e^{-2.07}\,2.07^x}{x!} = 392 * \frac{e^{-2.07}\,2.07^x}{x!}, \qquad x = 0,1,2,\ldots$$

x:	0	1	2	3	4	5	6
E_i	49	102	106	73	38	16	5
O_i	275	72	30	7	5	2	1

x:	O_i	E_i	$(O_i - E_i)$	$\frac{(O_i - E_i)^2}{E_i}$
0	275	49	226	1042.4
1	72	102	-30	8.823

2	30	106	-76	54.49
3	7	73	-66	59.67
4	5	37	-32	27.67
5	2	16	-14	12.25
6	1	5	-4	3.2

$$\chi^2 = \mathbf{1208.503}$$

Table value of $\chi^2_{0.05}$ with $n-1 = 7-1 = 6$ d.o.f is 5.99.

Conclusion:

Since $\chi^2 > \chi^2_{0.05}$, we reject null hypothesis.

OR

11. (b). (i). Sandal powder packed into packets by a machine. A random sample of 12 packets is drawn and their weight are found to be (in kg) 0.49, 0.48, 0.47, 0.48, 0.49, 0.50, 0.51, 0.49, 0.48, 0.50, 0.51 and 0.48. test if the average weight of the packing can be taken as 0.5 kg at 5% level of significance.

Solution:

x_i	0.49	0.48	0.47	0.48	0.49	0.5	0.51	0.49	0.48	0.5	0.51	0.48	$\sum x = 5.88$
$x_i - \bar{x}$	0	-0.01	-0.02	-0.01	0	0.01	0.02	0	-0.01	0.01	0.02	-0.01	
$(x_i - \bar{x})^2$	0	0.0001	0.0004	0.0001	0	0.0001	0.0004	0	0.0001	0.0001	0.0004	0.0001	$\sum (x_i - \bar{x})^2 = 0.0018$

$$\bar{x} = \frac{\sum x_i}{n} = \frac{5.88}{12} = 0.49$$

$$S = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{0.0018}{12-1}} = \sqrt{0.00016}$$

$$\therefore \quad S = 0.0128$$

Hence $n = 12$, $\bar{x} = 0.49$, $S = 0.0128$, $\mu = 0.5$

Null Hypothesis : $H_0 : \mu = 5$

Alternative Hypothesis : $H_1 : \mu \neq 5$

The test statistic is given by

$$t = \frac{\bar{x} - \mu}{\frac{S}{\sqrt{n}}} \quad \text{with } n-1 \text{ degrees of freedom}$$

$$t = \frac{0.49 - 0.5}{\frac{0.0128}{\sqrt{12}}} = -2.706$$

$$|t| = 2.706$$

The critical value for t for a two tailed test at 5% level of significance with $12 - 1 = 11$ d.o.f is 2.20.

Calculated value =2.706 and Tabulated value =2.20

$$|Calculated\ value| > Tabulated$$

$|2.706| > 2.20 \quad Reject\ H_0$

11. (b). (ii). A group of 10 rats fed on diet A and another group of 8 rats on died B, recorded the following increase in weight.

Diet A	5	6	8	1	12	4	3	9	6	10
Diet B	2	3	6	8	10	1	2	8		

Test the hypothesis that the samples have same from population with equal variances at 5% level of significance.

Solution:

x_i	5	6	8	1	12	4	3	9	6	10	$\sum x = 64$
$x_i - \bar{x}$	-1.4	-0.4	1.6	-5.4	5.6	-2.4	-3.4	2.6	-0.4	3.6	
$(x_i - \bar{x})^2$	1.96	0.16	2.56	29.16	31.36	5.76	11.56	6.76	0.16	12.96	$\sum (x_i - \bar{x})^2 = 102.4$
y_i	2	3	6	8	10	1	2	8			$\sum y = 40$
$y_i - \bar{y}$	-3	-2	1	3	5	-4	-3	3			
$(y_i - \bar{y})^2$	4	9	36	64	100	1	4	64			$\sum (y_i - \bar{y})^2 = 282$

$$\bar{x} = \frac{\sum x_i}{n_1} = \frac{64}{10} = 6.4 \qquad and \qquad \bar{y} = \frac{\sum y_i}{n_2} = \frac{40}{8} = 5$$

$$S_1 = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n_1 - 1}} = \sqrt{\frac{102.4}{10 - 1}} = 3.373 \qquad and \qquad S_2 = \sqrt{\frac{\sum (y_i - \bar{y})^2}{n_2 - 1}} = \sqrt{\frac{282}{8 - 1}} = 6.347$$

Null Hypothesis : $\boldsymbol{H_0 : \sigma_1^2 = \sigma_2^2}$

i.e., there is no significant difference between variances.

Alternative Hypothesis : $\boldsymbol{H_1 : \sigma_1^2 = \sigma_2^2}$ **(Two tailed test)**

The test statistic is given by

$$F = \frac{S_1^2}{S_2^2} = \frac{3.373}{6.347} = 0.53$$

The table value of $F_{0.05}(9,7) = 3.29 \quad [F_{0.05}(n_1 - 1\ ,\ n_2 - 1\ )]$

Calculated value =0.53 and Tabulated value =3.29

$|Calculated\ value\ | < Tabulated\ value \quad then\ Accept\ H_0$

$But \quad |0.53| < 3.29 \quad Accept \quad H_0$

i.e., there is no significant difference between variances.

OR

12. (b). (i). The following table shows the lives in hours of four batches of electric bulbs.

Batches						
1	1610	1610	1680	1700	1720	1800
2	1580	1640	1700	1750		
3	1460	1550	1620	1640	1740	1820
4	1510	1520	1570	1600	1680	

Perform an analysis of variance of these data and show that a significance test does not reject their homogeneity.

Solution: Let us subtract 1640 for simple calculations

Batches									Total
1	-30	-30	10	40	60	80	160		290
2	-60	0	0	60	110				110
3	-180	-90	-40	-20	0	20	100	180	-30
4	-130	-120	-110	-70	-40	40			-430
								Total	-60

Null Hypothesis: $H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$

i.e., the mean lives of four batches are homogeneous.

Alternative Hypothesis:

H_1: There is a significant difference among the four sample means.

Level of significance: $\propto = 0.05$, $N = 7 + 5 + 8 + 6 = 26$

$$Grand\ total \quad G = \sum\sum y_{ij} = -60$$

$$\therefore \ Correction\ factor = \frac{G^2}{N} = \frac{(-60)^2}{26} = 138$$

Total sum of squares $\quad SST = 195200 - 138 = 195062$

Between row sum of squares

$$SSR = \frac{(290)^2}{7} + \frac{(110)^2}{5} + \frac{(-30)^2}{8} + \frac{(-430)^2}{6} - C.F$$

$$SSR = 45364 - 138 = 45226$$

$$SSE = SST - SSR = 195062 - 45226 = 149836$$

ANOVA Table

Source of Variation	Degrees of Freedom	Sum of squares	Mean sum of squares	F-ratio
Between Rows	$k - 1 = 4 - 1 = 3$	45226	15075	$F = 2.21$
Error	$N - k = 26 - 4 = 22$	149836	6811	

The table value for $F_{(3,22)}$ at 5% level of significance is 3.06.

Conclusion:

Since the calculated value of F is less than the table value, the null hypothesis is accepted. That is the difference between the four mean lives are homogeneous.

12.. (b). (ii). Three verities of a crop tested in a randomized block design with four replications. The plot yield in pounds is as follows.

A 6   C 5   A 8   B 9

C 8	A 4	B 6	C 9
B 7	B 6	C 10	A 6

Analyse the experiment yield and state your conclusion.

Solution:

H_0: There is no significant difference between rows and columns.

Blocks	Crops			Total
	A	B	C	
1	6	7	8	21
2	4	6	5	15
3	8	6	10	24
4	6	9	9	24
Total	24	28	32	84

$$Correction\ factor = C.F = \frac{G^2}{N} = \frac{(84)^2}{12} = 588$$

$$SST = Total\ sum\ of\ squares = \{624\} - C.F$$

$$= 624 - 588 = 36$$

Between Column sum of squares

$$SSC = \frac{(\sum x_1)^2}{n_1} + \frac{(\sum x_2)^2}{n_1} + \frac{(\sum x_3)^2}{n_1} + \frac{(\sum x_4)^2}{n_1} + \frac{(\sum x_5)^2}{n_1} - C.F$$

$$SSC = \frac{(24)^2}{4} + \frac{(28)^2}{4} + \frac{(32)^2}{4} - 588 = 8$$

Between Row sum of squares

$$SSC = \frac{(\sum y_1)^2}{m_1} + \frac{(\sum y_2)^2}{m_1} + \frac{(\sum y_3)^2}{m_1} + \frac{(\sum y_4)^2}{m_1} - C.F$$

$$R_2 = \frac{(21)^2}{3} + \frac{(15)^2}{3} + \frac{(24)^2}{3} + \frac{(24)^2}{3} - 588 = 18$$

Error sum of squares

$$SSE = SST - SSR - SSC = 36 - 18 - 8 = 10$$

Degrees of freedom: $v_1 = c - 1 = 3 - 1 = 2,\quad v_2 = r - 1 = 4 - 1 = 3,\quad v_3 = v_1 * v_2 = 12$

ANOVA table for two-way classification				
Source of variation	Degrees of freedom	Sum of squares (SS)	Mean sum of squares (MS)	Variance Ration (F-Ratio)
B/W Column	2	SSC=8	$MSC = 2$	$F_1 = 3.6$
B/W Row	3	SSR=18	$MSR = 3$	$F_2 = 2.4$

Error	$2 * 3 = 6$	SSE=10	$MSE = 1.67$	

$$F_{0.05}(2,6) = 5.14 \quad and \quad F_{0.05}(3,6) = 4.76$$

Conclusion:

1. $F_1 < F_{0.05}(2,6)$. Hence we accept the null hypothesis. That is there is no difference between columns.
2. $F_2 < F_{0.05}(3,6) = 4.76$. Hence we accept the null hypothesis. That is there is some difference between Rows.

13. (a). (i). Find the largest Eigen value and the corresponding eigenvector of the matrix

$$A = \begin{pmatrix} 2 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \end{pmatrix}.$$

Solution: Let $X_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ be the initial vector.

Therefore,

$$A X_1 = \begin{pmatrix} 2 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \end{pmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 3 \\ 4 \end{bmatrix} = 4 \begin{bmatrix} 1 \\ 0.75 \\ 0.75 \\ 1 \end{bmatrix} = 4 X_2$$

$$A X_2 = A \begin{bmatrix} 1 \\ 0.75 \\ 0.75 \\ 1 \end{bmatrix} = \begin{bmatrix} 3.5 \\ 2.75 \\ 2.75 \\ 3.5 \end{bmatrix} = 3.5 \begin{bmatrix} 1 \\ 0.789 \\ 0.786 \\ 1 \end{bmatrix} = 3.5 X_3$$

$$A X_3 = A \begin{bmatrix} 1 \\ 0.789 \\ 0.786 \\ 1 \end{bmatrix} = \begin{bmatrix} 3.572 \\ 2.786 \\ 2.786 \\ 3.572 \end{bmatrix} = 3.572 \begin{bmatrix} 1 \\ 0.78 \\ 0.78 \\ 1 \end{bmatrix} = 3.572 X_4$$

$$A X_4 = A \begin{bmatrix} 1 \\ 0.78 \\ 0.78 \\ 1 \end{bmatrix} = \begin{bmatrix} 3.56 \\ 2.78 \\ 2.78 \\ 3.56 \end{bmatrix} = 3.56 \begin{bmatrix} 1 \\ 0.78 \\ 0.78 \\ 1 \end{bmatrix} = 3.56 X_5$$

$$A X_5 = A \begin{bmatrix} 1 \\ 0.78 \\ 0.78 \\ 1 \end{bmatrix} = \begin{bmatrix} 3.56 \\ 2.78 \\ 2.78 \\ 3.56 \end{bmatrix} = 3.56 \begin{bmatrix} 1 \\ 0.78 \\ 0.78 \\ 1 \end{bmatrix} = 3.56 X_6$$

$\therefore$ The dominant **Eigen value** $= 3.56$.

Corresponding **Eigen vector** is $\begin{bmatrix} 1 \\ 0.78 \\ 0.78 \\ 1 \end{bmatrix}$.

13. (a). (ii). Find the inverse by Gauss Jordan method of $A = \begin{pmatrix} 1 & 1 & 1 \\ 4 & 3 & 1 \\ 3 & 5 & 3 \end{pmatrix}$.

Solution:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 1 \\ 3 & 5 & 3 \end{bmatrix}$$

We know that $[A, I] = [I, A^{-1}]$

Now, $[A, I] = \begin{bmatrix} 1 & 1 & 1 & \vdots & 1 & 0 & 0 \\ 4 & 3 & 1 & \vdots & 0 & 1 & 0 \\ 3 & 5 & 3 & \vdots & 0 & 0 & 1 \end{bmatrix}$

Now, we need to make $[A. I]$ as a diagonal matrix.

Fix the first row, change second and third row by using first row.

$$[A, I] \sim \begin{bmatrix} 1 & 1 & 1 & \vdots & 1 & 0 & 0 \\ 0 & -1 & -3 & \vdots & -4 & 1 & 0 \\ 0 & 2 & 0 & \vdots & -3 & 0 & 1 \end{bmatrix} \quad \begin{matrix} R_2 \Leftrightarrow R_2 - 4\,R_1 \\ R_3 \Leftrightarrow \ R_3 - 3\,R_1 \end{matrix}$$

Fix the first row & second row, change third row by using second row.

$$[A, I] \sim \begin{bmatrix} 1 & 1 & 1 & \vdots & 1 & 0 & 0 \\ 0 & -1 & -3 & \vdots & -4 & 1 & 0 \\ 0 & 0 & -6 & \vdots & -11 & 2 & 1 \end{bmatrix} \quad R_3 \Leftrightarrow \ R_3 + 2\,R_2$$

Fix the third row, change first and second row by using third row.

$$[A, I] \sim \begin{bmatrix} -6 & -6 & 0 & \vdots & 5 & -2 & -1 \\ 0 & 6 & 0 & \vdots & -9 & 0 & 3 \\ 0 & 0 & -6 & \vdots & -11 & 2 & 1 \end{bmatrix} \quad \begin{matrix} R_1 \Leftrightarrow -6\,R_1 - 1\,R_3 \\ R_2 \Leftrightarrow -6\,R_2 - (-3)R_3 \end{matrix}$$

$$[A, I] \sim \begin{bmatrix} -6 & 0 & 0 & \vdots & -4 & -2 & 2 \\ 0 & 6 & 0 & \vdots & -9 & 0 & 3 \\ 0 & 0 & -6 & \vdots & -11 & 2 & 1 \end{bmatrix} \quad R_1 \Leftrightarrow \ R_1 + R_2$$

$$[A, I] \sim \begin{bmatrix} 1 & 0 & 0 & \vdots & -4/-6 & -2/-6 & 2/-6 \\ 0 & 1 & 0 & \vdots & -9/6 & 0/6 & 3/6 \\ 0 & 0 & 1 & \vdots & -11/-6 & 2/-6 & 1/-6 \end{bmatrix} \quad \begin{matrix} R_1 \Leftrightarrow R_1\,/-20 \\ R_2 \Leftrightarrow R_2\,/20 \\ R_3 \Leftrightarrow R_3\,/-10 \end{matrix}$$

$$A^{-1} = \begin{bmatrix} 2/3 & 1/3 & -1/3 \\ -3/2 & 0 & 1/2 \\ 11/6 & -1/3 & -1/6 \end{bmatrix}$$

OR

13. (b). (i). Using Gauss Elimination method solve the system $3.15x - 1.96y + 3.85z = 12.95$, $2.13x + 5.12y + 2.89z = -8.61$, $5.92x + 3.05y + 2.15z = 6.88$.

Solution:

The given system is equivalent to $A\,X = B$

$$\begin{bmatrix} 3.15 & -1.96 & 3.85 \\ 2.13 & 5.12 & -2.89 \\ 5.92 & 3.05 & 2.15 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12.95 \\ -8.61 \\ 6.88 \end{bmatrix}$$

$$Here\ \ [A, B] = \begin{bmatrix} 3.15 & -1.96 & 3.85 & 12.95 \\ 2.13 & 5.12 & -2.89 & -8.61 \\ 5.92 & 3.05 & 2.15 & 6.88 \end{bmatrix}$$

Now, we need to make A as an upper triangular matrix.

Fix the first row, change second and third row by using first row.

$$[A, B] \sim \begin{bmatrix} 3.15 & -1.96 & 3.85 & 12.95 \\ 0 & 20.3028 & -17.304 & -54.705 \\ 0 & 21.2107 & -16.0195 & -54.92 \end{bmatrix} \quad \begin{matrix} R_2 \Leftrightarrow 3.15R_2 - 2.13\,R_1 \\ R_3 \Leftrightarrow 3.15\,R_3 - 5.92\,R_1 \end{matrix}$$

Fix the first & second row, change the third row by using second row.

$$[A,B] \sim \begin{bmatrix} 3.15 & -1.96 & 3.85 & 12.95 \\ 0 & 20.3028 & -17.304 & -54.705 \\ 0 & 0 & 41.7892 & 43.8398 \end{bmatrix} \quad R_3 \Leftrightarrow 20.028\, R_3 - 21.2107\, R_2$$

This is an upper triangular matrix. From the above matrix we have

$$41.7892\, z = 43.8398 \quad \Longrightarrow \quad z = 1.049$$

$$20.3028\, y - 17.304\, z = -54.705 \quad \Longrightarrow 20.3028\ \ y - 17.304(1.049) = \ -54.705$$

$$\Longrightarrow\ 20.3028\ \ y = \ -36.5531$$

$$\Longrightarrow \quad y = -1.8$$

$$3.15x - 1.96y + 3.85z = 12.95$$

$$3.15x - 1.96(-1.8) + 3.85(1.049) = 12.95$$

$$3.15x = 5.38335 \quad \Longrightarrow x = 1.709$$

Hence the solution is $\boldsymbol{x = 1.709}$, $\boldsymbol{y = -1.8}$ $\boldsymbol{and}$ $\boldsymbol{z = 1.049.}$

13. (b). (ii). Solve the following system of equations by Gauss Jacobi and Siedal method, correct to three decimal places $8x - 3y + 2z = 20$, $4x + 11y - z = 33$, $4x + 11y - z = 33$.

Solution:

$$8x - 3y + 2z = 20$$

$$4x + 11y - z = 33$$

$$4x + 11y - z = 33$$

Since the diagonal elements are dominant in the coefficient matrix, we rewrite x, y, z as follows

$$x = \frac{1}{8}(20 + 3y - 2z)\,, \qquad y = \frac{1}{11}(33 - 4x + z), \qquad z = \frac{1}{12}(35 - 6x - 3y)$$

Gauss Jacobi Method:

Let the initial values be $\boldsymbol{x = 0, y = 0, z = 0}$

We form the Iterations in the table

Iteration	x	y	z
1	2.5	3	2.9167
2	2.8958	2.3561	0.9167
3	3.1544	2.0303	0.8797
4	3.0414	1.9329	0.8319
5	3.0169	1.9697	0.9127
6	3.0105	1.9859	0.9158
7	3.0158	1.9885	0.9149
8	3.017	1.9865	0.9116
9	3.0168	1.9858	0.9115
10	3.0168	1.9858	0.9117

Hence the solution is $\boldsymbol{x} = 3.0168$, $\boldsymbol{y} = 1.9858$ $\boldsymbol{and}$ $\boldsymbol{z} = 0.9117$.

Gauss Siedal Method:

Let the initial values be $\boldsymbol{y = 0, z = 0}$

We form the Iterations in the table

Iteration	x	y	z
1	2.5	2.0909	1.1439
2	2.9981	2.0137	0.9142
3	3.0266	1.9825	0.9077

4	3.0165	1.9856	0.9120
5	3.0166	1.986	0.9119
6	3.0168	1.9859	0.9118
7	3.0168	1.9859	0.9118

Hence the solution is $\boldsymbol{x} = 3.0168$, $\boldsymbol{y} = 1.9859$ $\boldsymbol{and}$ $\boldsymbol{z} = 0.9118$.

14. (a). (i). Write the Newton's method formula and using it obtain $f(x)$ as a polynomial in power of $(x - 5)$ form the given table.

x:	0	2	3	4	5	6
f(x):	4	36	58	112	466	922

Solution:

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$	$\Delta^5 f(x)$
0	4					
		16				
2	36		2			
		22		3.5		
3	58		16		8.2334	
		54		44.66		-4.609
4	112		150		-19.418	
		354		-33		
5	466		51			
		456				
6	922					

By Newton's divided difference interpolation formula

$$f(x) = f(x_0) + (x - x_0)\Delta f(x) + (x - x_0)(x - x_1)\Delta^2 f(x) + (x - x_0)(x - x_1)(x - x_2)\Delta^3 f(x)$$
$$+ (x - x_0)(x - x_1)(x - x_2)(x - x_3)\Delta^4 f(x) + (x - x_0)(x - x_1)(x - x_2)(x - x_3)(x - x_4)\Delta^5 f(x)$$

$$\therefore \quad f(x) = 4 + (x - 0)\,(16) + (x - 0)(x - 2)\,(2) + (x)(x - 2)(x - 3)\,(3.5) + (x)(x - 2)(x - 3)(x - 4)(8.233)$$
$$+ (x)(x - 2)(x - 3)(x - 4)(x - 5)(0 - 4.609)$$

14. (a). (ii). A rod is rotating in a plane. The following table gives the angle θ (in radians) through which the rod has turned for various values of time t (seconds). Calculate the angular velocity and angular acceleration of the rod at $t = 0.6$ $seconds$.

T:	0	0.2	0.4	0.6	0.8	1.0
θ:	0	0.12	0.49	1.12	2.02	3.20

Solution:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	0				
		0.12			
0.2	0.12		0.25		
		0.37		0.1	
0.4	0.49		0.26		0
		0.63		0.1	
0.6	0.12		0.27		0
		0.9		0.1	
0.8	2.02		0.28		
		1.18			
1.0	3.20				

To find $y\ at\ x = 0.6$, $[Nearer\ to\ x_n]$. Here $h = 0.2$

$$Velocity = \frac{d\theta}{dt} = \frac{1}{h}\left[\nabla y_n + \frac{2V+1}{2}\nabla^2 y_n + \frac{3V^2+6V+2}{6}\nabla^3 y_n + \ \dots\right]$$

$$\boldsymbol{V} = \frac{x - x_n}{h} = \frac{0.6 - 1.0}{0.2} = \mathbf{-2}$$

$$\frac{d\theta}{dt} = \frac{1}{0.2}\left[1.18 + \frac{2(-2)+1}{2}(0.28) + \frac{3(-2)^2+6(-2)+2}{6}(0.1) + \ \dots\right]$$

$$= \frac{1}{0.2}\left[\, 1.18 - 0.42 + 0.0033 \,\right]$$

$$\boldsymbol{Velocity = \frac{d\theta}{dt} = 3.8165}$$

$$Accelaration \ \ \frac{d^2\theta}{dt^2} = \frac{1}{h^2}\left[\ \nabla^2 y_n + (V+1)\,\nabla^3 y_n + \ \dots\right]$$

$$\frac{d^2\theta}{dt^2} = \frac{1}{0.2^2}\left[0.28 + (-2+1)\,(0.1) + \ \dots\right]$$

$$= \frac{1}{0.2^2}\left[0.28 - 0.01\right]$$

$$\boldsymbol{Accelaration = \frac{d^2\theta}{dt^2} = 6.75}$$

OR

14. (b). (i) Write Trapezoidal rule and Simpson's rule for evaluation os $I = \int_{x_0}^{x_n} f(x)\,dx$, evaluate $I = \int_0^6 \frac{1}{1+x}\,dx$ using Trapezoidal rule, Simpson's rule. Also check up by direct integration.

Solution:

Here $y(x) = \frac{1}{1+x^2}$. Range $= b - a = 6 - 0 = 6$

So we divide 6 equal intervals with $h = \frac{6}{6} = 1$. We form a table

x :	0	1	2	3	4	5	6
$y = \frac{1}{1+x^2}$:	1	0.500	0.200	0.100	0.058824	0.038462	0.27027

Trapezoidal rule

$$\int_0^6 \frac{1}{1+x^2}\,dx = \frac{h}{2}\left[(y_0 + y_6) + 2(y_1 + y_2 + y_3 + y_4 + y_5)\right]$$

$$= \frac{1}{2}\left[(1 + 0.027027) + 2(0.5 + 0.2 + 0.1 + 0.058824 + 0.038462)\right]$$

$$\int_0^6 \frac{1}{1+x^2}\,dx = 1.4107995.$$

(2). **By Simpson's 1/3 rule** :

$$\int_0^6 \frac{1}{1+x^2}\,dx = \left(\frac{h}{3}\right)\left[(y_0 + y_6) + 2(y_1 + y_3 + y_5) + 4(y_2 + y_4)\right]$$

$$= \frac{1}{3}\left[(1 + 0.027027) + 2(0.2 + 0.058824) + 4(0.5 + 0.1 + 0.038462)\right]$$

$$\int_0^{\pi} \sin x \; dx = 1.36617433.$$

(3). **By Simpson's 3/8 rule** :

$$\int_a^b f(x)\,dx = \frac{3h}{8}\,[(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5 + ..) + 2(y_3 + y_6 + ..)]$$

$$\int_0^6 \frac{1}{1+x^2}\,dx = \frac{3h}{8}\,[(y_0 + y_6) + 3(y_1 + y_2 + y_4 + y_5) + 2(y_3)]$$

$$= \frac{3}{8}\,[(1 + 0.027027) + 3(0.5 + 0.2 + 0.058824 + 0.038462) + 2(0.1)]$$

$$\int_0^6 \frac{1}{1+x^2}\,dx = 1.357081875.$$

By Actual Integration:

$$\int_0^6 \frac{1}{1+x^2}\,dx = [\tan^{-1} x]_0^6 = \tan^{-1} 6 - \tan^{-1} 0 = 1.40564765$$

14. (b). (ii).

(1). Given the following data, find $y'(6)$ and the maximum value of $y(5)$

X	0	2	3	4	7	9
y:	4	26	58	112	465	922

Solution:

We form the divided difference table (Since Unequal intervals)

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
0	**4**			
		11		
2	26		**7**	
		32		**1**
3	58		11	
		54		1
4	112		16	
		118		1
7	465		22	
		228		
9	922			

By Newton's divided difference interpolation formula

$$f(x) = f(x_0) + (x - x_0)f(x_0, x_1) + (x - x_0)(x - x_1)f(x_0, x_1, x_2) + (x - x_0)(x - x_1)(x - x_2)f(x_0, x_1, x_2, x_3)$$

$$\therefore \quad f(x) = 4 + (x - 0)\,11 + (x - 0)(x - 2)\,7 + (x - 0)(x - 2)(x - 3)1$$

$$= 4 + 11x + (x^2 - 2x)\,7 + (x^2 - 2x)(x - 3)$$

$$= 4 + 11x + 7x^2 - 14x + x^3 - 2x^2 - 3x^2 + 6x$$

$$f(x) = x^3 + 2x^2 + 3x + 4$$

$$f'(x) = 3x^2 + 4x + 3$$

$$f'(6) = 3(6)^2 + 4(6) + 3 = 135$$

To find Maximum:

$f'(x) = 0 \qquad \Rightarrow \quad 3x^2 + 4x + 3 = 0$

The roots are imaginary, there is no extremum.

(2). Using Lagrange's formula of interpolation find $y(9.5)$ given,

X	7	8	9	10
y:	3	1	1	9

Solution:

x :	7 (x_0)	8 (x_1)	9 (x_2)	10 (x_3)
$f(x)$:	3 (y_0)	1 (y_1)	1 (y_2)	9 (y_3)

Lagrange's interpolation formula, we have

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)}\, y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)}\, y_1$$

$$+\frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)}\, y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)}\, y_3$$

$$+\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)}\, y_4$$

$$f(x) = \frac{(x-8)(x-9)(x-10)}{(7-8)(7-9)(7-10)}\,(3) + \frac{(x-7)(x-9)(x-10)}{(8-7)(8-9)(8-10)}\,(1)$$

$$+\frac{(x-7)(x-8)(x-10)}{(9-7)(9-8)(9-10)}\,(1) + \frac{(x-7)(x-8)(x-9)}{(10-7)(10-8)(10-9)}\,(9)$$

To fond $y(9.5)$: $(x = 9.5)$:

$$y = f(2) = \frac{(9.5-8)(9.5-9)(9.5-10)}{(7-8)(7-9)(7-10)}\,(3) + \frac{(9.5-7)(9.5-9)(9.5-10)}{(8-7)(8-9)(8-10)}\,(1)$$

$$+\frac{(9.5-7)(9.5-8)(9.5-10)}{(9-7)(9-8)(9-10)}\,(1) + \frac{(9.5-7)(9.5-8)(9.5-9)}{(10-7)(10-8)(10-9)}\,(9)$$

$$y = f(9.5) = 0.1875 - 0.3125 + 0.9375 + 0.3125$$

$$y = f(9.5) = 1.125.$$

15. (a). (i). Given $\frac{dy}{dx} = \frac{1}{2}(1+x^2)y^2$ and $y(0) = 1$. Evaluate $y(0.4)$ by Milne's predicator and corrector formula.

Solution: Given

$$y' = f(x,y) = \frac{dy}{dx} = \frac{1}{2}(1+x^2)\, y^2 \quad , \quad x_0 = 0, \qquad y_0 = 1$$

To find $y(0.1)$, $y(0.2)$ *and* $y(0.3)$ by Euler's method.

The Euler's formula is

$$\boldsymbol{y_{n+1}(x_n + h) = y_n + h\,[f(x_n, y_n)] \quad , \quad n = 0, 1, 2, \ldots \qquad \ldots\ldots\ldots (1)}$$

To find $y(0.1)$: Put $n = 0$, equation (1) becomes

$$y_1(x_0 + h) = y_0 + h\,[f(x_0, y_0)]$$

$$\therefore \quad y_1(0+0.1) = 1 + (0.1)\ [f(0,\ \ 1\ )]$$

$$= 1 + (0.1)\ \left[\frac{1}{2}(1+0^2)(1)^2\right] = 1.05$$

$$\boldsymbol{y_1(0.1) = 1.05} \qquad \Longrightarrow\ x_1 = \mathbf{0.1}\ \ \&\ \boldsymbol{y_1 = 1.05}$$

To find $\boldsymbol{y(0.2)}$ **:** Put $n = 1,$ equation (1) becomes

$$y_1(x_1 + h) = y_1 +\ h\ [f(x_1, y_1)]$$

$$\therefore \quad y_1(0.1+0.1) = 1.05 + (0.1)\ [f(0.1,\ \ 1.05\ )]$$

$$= 1.05 + (0.1)\ \left[\frac{1}{2}(1+0.1^2)(1.05)^2\right] = 1.1056$$

$$\boldsymbol{y_1(0.2) = 1.1057} \qquad \Longrightarrow\ x_2 = \mathbf{0.2}\ \ \&\ \boldsymbol{y_2 = 1.1056}$$

To find $\boldsymbol{y(0.3)}$ **:** Put $n = 2,$ equation (1) becomes

$$y_3(x_2 + h) = y_2 +\ h\ [f(x_2, y_2)]$$

$$\therefore \quad y_3(0.2+0.1) = 1.1056 + (0.1)\ [f(0.2,\ \ 1.1056\ )]$$

$$\boldsymbol{y_1(0.3) = 1.1691} \qquad \Longrightarrow\ x_3 = \mathbf{0.3}\ \ \&\ \boldsymbol{y_3 = 1.1693}$$

$y(0) = 1$	$x_0 = 0$	$y_0 = 1$	
$y(0.1) = 1.05$	$x_1 = 0.1$	$y_1 = 1.05$	$y_1' = 0.5568$
$y(0.2) = 1.1057$	$x_2 = 0.2$	$y_2 = 1.1056$	$y_2' = 0.6357$
$y(0.3) = 1.1693$	$x_3 = 0.3$	$y_3 = 1.1693$	$y_3' = 0.7452$

The Milne's Predictor formula is

$$y_{n+1,P}(x_n + h) = y_{n-3} + \frac{4h}{3}[2\ y_{n-2}' - y_{n-1}' + 2\ y_n'] \quad \dots\dots(1)$$

Put n=3 in equation (1), we have

$$y_{4,P}(x_3 + h) = y_0 + \frac{4h}{3}[2\ y_1' - y_2' + 2\ y_3'] \qquad \dots\dots(2)$$

Equation (2) becomes

$$y_{4,P}(0.3+0.1) = 1 + \frac{4 * 0.1}{3}[2\ (0.5568) - 0.6357 + 2\ (0.7452)]$$

$$\boldsymbol{y_{4,P}(0.4)} \ \ = \mathbf{1.2624} \quad [\boldsymbol{y(x_4) = y_4},\ \ \boldsymbol{x_4 = 0.4}\ \ \&\ \ \boldsymbol{y_4 = 1.2624}\ ]$$

The Milne's Corrector formula is

$$y_{n+1,C}(x_n + h) = y_{n-1} + \frac{h}{3}[\ y_{n-1}' + 4y_n' +\ y_{n+1}'] \quad \dots\dots(3)$$

Put $\boldsymbol{n = 3}$ in equation (3), we have

$$y_{4,C}(x_3+h) = y_2 + \frac{h}{3}[\ 0.6357 + 4(0.7452) +\ 0.9243] \quad \ldots\ldots(4)$$

$x_4 = 0.4$	$y_4 = 1.3127$	$y_4{}' = \frac{1}{2}[1+0.4^2]\ (1.2624)^2$	$y_4{}' = 0.9243$

Equation (4) becomes

$$y_{4,C}(0.3+0.1) = 1.1056 + \frac{0.1}{3}[\ y_2' + 4y_3' +\ y_4']$$

$$y_{4,C}(0.4) \ = 1.25706 \quad [y(x_4) = y_4, \quad x_4 = 0.4 \ \ \& \ \ y_4 = 1.2570\ ]$$

15. (a). (ii) Solve the BVP $u'' = xu,\ u(0) + u'(0) = 1,\ \ u(1) = 1,\ h = \frac{1}{3}$, use the second order method.

Solution:

The finite difference equation is

$$\frac{y_{i+1} + y_{i-1} - 2y_i}{h^2} = x_i\, y_i$$

$$y_{i+1} + y_{i-1} - 2y_i - h^2\, x_i\, y_i$$

$$y_{i+1} + y_{i-1} - (2 + h^2 x_i) y_i = 0$$

$$y_{i+1} + y_{i-1} - \left(2 + \frac{x_i}{9}\right) y_i = 0 \quad ---(1)$$

Put $i = 1,2,3$, we get

$$y_2 + y_0 - \left(2 + \frac{x_1}{9}\right) y_1 = 0 \quad \Rightarrow \quad y_2$$

OR

15. (b). (i). Given $y'' + xy' + y = 0,\ y(0) = 1,\ \ y'(0) = 0$ find the value of $y(0.1)$ by using Runge-Kutta method of fourth order.

Solution:

$$Given\ \ y'' + xy' + y = 0\ \ i.e.,\ \ \frac{d^2y}{dx^2} = -x\, y' - y,\ \ y(0) = 1 \qquad \Rightarrow x_0 = 0,\ \ y_0 = 1$$

Put $y' = z$, we get $\frac{d^2y}{dx^2} = f(x,y,z) = -x\, z - y\ ,\ \ z_0 = y_0' = 0 \quad Since \ \ y_0 = 1$

$$\frac{dy}{dx} = z = f_1(x,y,z) \quad and \quad \frac{dz}{dx} = f_2(x,y,z) = -xz - y$$

The algorithm for fourth order R-K method is

To find $y(0.1)$:

$$k_1 = h\, f_1(x_0, y_0, z_0) = 0.1\ f_1(0, 1,0\ ) = 0.1\ [0] = 0$$

$$\boldsymbol{k_1 = 0}$$

$$l_1 = hf_2(x_0, y_0, z_0) = 0.1\ f_1(0,1,0\ ) = 0.1(-1) = -0.1$$

$$k_2 = h\, f_1\left(x_0 + \frac{h}{2},\ y_0 + \frac{k_1}{2},\ \ z_0 + \frac{l_1}{2}\right) = 0.1\ f_1\ \left(0 + \frac{0.1}{2},\ \ 1 + \frac{0.1}{2}\ ,0 - \frac{0.1}{2}\right) = 0.1 f_1(0.05,1,-0.05)$$

$\boldsymbol{k_2 = -0.005}$

$$l_2 = h\, f_2\left(x_0 + \frac{h}{2},\ y_0 + \frac{k_1}{2},\ \ z_0 + \frac{l_1}{2}\right) = 0.1\ f_2\left(0 + \frac{0.1}{2},\ \ 1 + \frac{0.1}{2}\ , 0 - \frac{0.1}{2}\right) = 0.1 f_2(0.05, 1, -0.05)$$

$l_2 = -0.09975$

$$k_3 = h\, f_1\left(x_0 + \frac{h}{2},\ y_0 + \frac{k_2}{2},\ \ z_0 + \frac{l_2}{2}\right) = 0.1\ f_1\ (0.05,\ \ 0.9975,\ \ -0.0499\ )$$

$\boldsymbol{k_3 = -0.0049}$

$$l_3 = h\, f_2\left(x_0 + \frac{h}{2},\ y_0 + \frac{k_2}{2},\ \ z_0 + \frac{l_2}{2}\right) = 0.1\ f_2\ (0.05,\ \ 0.9975,\ \ -0.0499\ )$$

$l_3 = -0.0995$

$$k_4 = h\, f_1(x_0 + h,\ y_0 + k_2,\ \ z_0 + l_2\ ) = 0.1\ f_1\ (0.1,\ \ 0.995511,\ \ \ \ \ -0.0995)$$

$\boldsymbol{k_4 = -0.00995}$

$$l_4 = h\, f_2(x_0 + h,\ y_0 + k_2,\ \ z_0 + l_2\ ) = 0.1\ f_2\ (0.1,\ \ 0.995511,\ \ \ \ \ -0.0995)$$

$l_4 = 0.9950$

$$\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = \frac{1}{6}(0 + 2(\mathbf{-0.005}) + 2(\mathbf{-0.0049}) - \mathbf{0.00995})$$

$\boldsymbol{\Delta y = -0.004958}$

$$y(x_0 + h) = y(x_0) + \Delta y = y_0 + \Delta y = 1 - 0.004958$$

$\boldsymbol{y(0.1) = 0.99504}$

15. (b). (ii). Using Runge-Kutta method of order four solve $y' = \frac{y^2 - x^2}{y^2 + x^2}$, given $y(0) = 1$ at $x = 0.2,\ 0.4$ correct to four decimal places.

Repeated in I question, No. 15. (a). (i).