STATISTICS AND NUMERICAL METHODS

QUESTION II NOVEMBER / DECEMBER 2010

1. What are the parameters and statistics in sampling?

Solution:

To avoid verbal confusion with the statistical constant of the population, namely $\boldsymbol{mean}\ \mu$, $variance\ \sigma^2$ which are usually referred to as parameters. Statistical measures computed from sample observations alone. E.g. $\boldsymbol{mean}\ (\bar{x}), variance\ (s^2),\ etc.$ are usually referred to as statistic.

2**. Write any two applications of 'ψ^2 ' test.**

Solution:

'$\boldsymbol{\psi^2}$' $test$ is used to test whether differences between observed and expected frequencies or significances.

3. Compare One way classification modal with Two way classification modal.

Solution:

	One way	Two way
1	We cannot test two sets of Hypothesis	Two sets of hypothesis can tested.
2	Data are classified according to one factor	Data are classified according to two different factor.

4. What is meant by Latin square?

Solution:

The n treatments are then allocated at random to these rows and columns in such a way that every treatment occurs once and only once in each row and in each column. Such a layout is known as $n \times n$ Latin square design.

5. Write the convergence condition and order of convergence for Newton- Raphson –method.

Solution:

The Criterion for convergence of Newton- Raphson -method is

$$|f(x)\ f''(x)| < |\ f'(x)|^2 \quad in\ the\ interval\ considered.$$

The order of convergence of Newton- Raphson –method is 2.

6. **Compare Gauss Jacobi with Gauss Jordan.**

Solution:

	Gauss – Jordan	**Gauss – Jacobi method**
1	It gives exact value	Convergence rate is slow
2	Simple, it takes less time	Indirect method
3	This method determines all the roots at the same time.	This method determines only one root at a time.

7. **Create a forward difference table for the following data and state the degree of polynomial for the same.**

Solution:

x	$Y = f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$
0	-1	1	2	0

1	0	3	2	
2	3	5		
3	8			

8. Compare Simpson's $\frac{1}{3}$ rule with Trapezoidal method.

Solution:

	Simpson's Rule	**Trapezoidal rule**
1	Must accurate	Least accurate
2	Interval of integration must be divided into even number of subintervals	Can be divide into any number of subintervals.

9. Using Taylor series find $y(0.1)$ for $\frac{dy}{dx} = 1 - y,\ \ y(0) = 0.$

Solution: $Given \quad \frac{dy}{dx} = 1 - y \quad \& \quad y(0) = 0 \implies x_0 = 0,\ y_0 = 0$

Taylor series formula is

$$y(x) = y_0 + \frac{(x - x_0)}{1\,!} y_0' + \frac{(x - x_0)^2}{2\,!} y_0'' + \frac{(x - x_0)^3}{3\,!} y_0''' + \frac{(x - x_0)^4}{4\,!} y_0^{iv} + \cdots \quad \ldots\ldots(1)$$

$y' = 1 - y$	$y_0' = 1 - 0 = 1$
$y'' = -y'$	$y_0'' = -1$
$y''' = -y''$	$y_0''' = -1$

Therefore equation (1) becomes,

$$y(x) = y_0 + \frac{(x - x_0)}{1\,!} y_0' + \frac{(x - x_0)^2}{2\,!} y_0'' + \frac{(x - x_0)^3}{3\,!} y_0''' +$$

$$= 1 + \frac{(0.1 - 0)}{1} (1) + \frac{(0.1 - 0)^2}{2} (-1) + \frac{(0.1 - 0)^3}{6} (1)$$

$\therefore \quad \boldsymbol{y(0.1) = 0.0952}$

10. Solve $y_{x+2} - 4\, y_x = 0.$

Solution:

Given $y_{x+2} - 4\, y_x = 0$

$i.e.,\ (E^2 - 4) y_x = 0$

$m^2 - 4 = 0$

$m = \pm 2.$

$y_x = A(2)^x + B(-2)^x.$

Part - B

11. (a). (i). A machine puts out 16 imperfect articles in a sample of 500. After the machine is overhauled, it puts out 3 imperfect articles in a batch of 100. Has the machine been improved?

Solution:

$Given\ that\ \ n_1 = 500,\ \ x_1 = 16 \quad and \quad n_2 = 100,\ \ x_2 = 3$

$p_1 = Prop\ before\ service \quad p_1 = \frac{x_1}{n_1} = \frac{16}{500} = 0.032 \ \ \&$

$q_1 = 1 - p_1 = 1 - 0.03 = 0.968$

$p_2 = Prop\ after\ service \quad p_2 = \frac{x_2}{n_2} = \frac{3}{100} = 0.032$ &

$q_2 = 1 - p_1 = 1 - 0.03 = 0.968$

Null Hypothesis:

$H_0 : P_1 = P_2 \quad i.e., \quad the\ machine\ has\ not\ improved$

Alternative hypothesis:

$H_1 : P_1 \neq P_2 \quad i.e.,$

Here Population Proportion P is not known.

$\therefore \quad P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$

$P = \frac{500 * 0.032 + 100 * 0.03}{500 + 100} = 0.03167$

$\therefore \quad P = 0.032 \quad and \quad Q = 1 - P = 1 - 0.032 = 0.968$

$The\ test\ statistic\ is\ given\ by\ z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

$$z = \frac{0.032 - 0.03}{\sqrt{[0.032 * 0.968]\left(\frac{1}{500} + \frac{1}{100}\right)}} = \frac{0.002}{\sqrt{[0.030976](0.012)}} = \frac{0.002}{0.0193}$$

$z = 0.1037$

At 5% level of significance, the table value for $z_\propto = 1.96$.

$$|calculated\ value\ | \leq tabulated\ value \quad \Rightarrow \quad Accept\ H_0$$

$$|0.1037| \leq 1.645 \quad \Rightarrow \quad Accept\ \ H_0$$

Conclusion: We accept the null hypothesis. That is the machine has improved after service.

11 (a). (ii). Examine whether the difference in the variability in yields is significant at 5% level of significance, for the following.

	Set of 40 plots	Set of 60 plots
Mean yield per plot	1256	1243
S.D per plot	34	28

Solution:

Given that $\overline{x_1} = 1256, \quad \overline{x_2} = 1243, \quad s_1 = 34, \quad s_2 = 28, \quad n_1 = 40 \quad and \quad n_2 = 60$

Null Hypothesis: $H_0 : \mu_1 = \mu_2$ i.e., there is no difference b/w two sets of yields.

Null Hypothesis: $H_0 : \mu_1 \neq \mu_2$

The test statistic is given by

$$z = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{1256 - 1243}{\sqrt{\frac{34^2}{40} + \frac{28^2}{60}}} = \frac{13}{\sqrt{41.966}} = 2.00$$

At 5% significance level the tabulated value for $z_\propto$ is 1.96.

$$|Calculated\ value\ | > Tabulated\ value$$

$$|2.00| > 1.96 \quad So\ we\ Reject\ H_0$$

Conclusion:

$|z| > z_{\propto}$, we reject the Null Hypothesis. That is the two plots of yields differs significantly.

11. (b). (i). Test if the difference in the means is significantly for

Sample I	76	68	70	43	94	68	33	
Sample II	40	48	92	85	70	76	68	22

Solution:

x_i	76	68	70	43	94	68	33		$\sum x = 452$
$x_i - \bar{x}$	11.4	3.4	5.4	-21.6	29.4	3.4	-31.6		
$(x_i - \bar{x})^2$	129.96	11.56	29.16	466.56	864.36	11.56	998.56		$\sum (x_i - \bar{x})^2 = 2511.72$
y_i	40	48	92	85	70	76	68	22	$\sum y = 501$
$y_i - \bar{y}$	-22.6	-14.6	29.4	22.4	7.4	13.4	5.4	-40.6	
$(y_i - \bar{y})^2$	510.76	213.16	864.36	501.76	54.76	179.56	29.16	1648.36	$\sum (y_i - \bar{y})^2 = 4001.88$

$$\bar{x} = \frac{\sum x_i}{n_1} = \frac{452}{7} = 64.6 \qquad and \qquad \bar{y} = \frac{\sum y_i}{n_2} = \frac{501}{8} = 62.6$$

$$\sigma_1 = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n_1 - 1}} = \sqrt{\frac{2511.72}{7 - 1}} \qquad and \qquad \sigma_2 = \sqrt{\frac{\sum (y_i - \bar{y})^2}{n_2 - 1}} = \sqrt{\frac{4001.88}{8 - 1}}$$

$$\therefore \quad \sigma_1 = \sqrt{418.62} = 20.46 \qquad and \qquad \sigma_2 = \sqrt{571.69} = 23.91$$

Hence $n_1 = 7, \quad n_2 = 8, \quad \overline{x_1} = 64.6, \quad \overline{x_2} = 62.6 \quad \sigma_1 = 20.46\,, \ \sigma_2 = 8.266$

Null Hypothesis : $\boldsymbol{H_0 : \mu_1 = \mu_2}$

i.e., there is no significant difference between two samples. groups.

Alternative Hypothesis : $\boldsymbol{H_1 : \mu_1 \neq \mu_2}$

The test statistic is given by

$$t = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)}} \qquad with\ (n_1 + n_2 - 2)\ degrees\ of\ freedom$$

$$t = \frac{64.6 - 62.6}{\sqrt{\frac{(20.46)^2}{7} + \frac{(8.266)^2}{8}}} = \frac{2}{\sqrt{68.342}} = 0.242$$

$$|t| = 0.242$$

The table value for t at 5% level of significance with $7 + 8 - 2 = 13$ degrees of freedom is 2.16.

Calculated value =0.242 and Tabulated value =2.16

$$|Calculated\ value\,| < Tabulated\ value \quad then\ Accept\ \ H_0$$

$$|0.57| < 1.76, \quad we\ \ Accept\ \ H_0$$

Conclusion:

$|t_{\propto}| < t$, we Accept H_0. That is there is no significant difference between the two sample means.

11. (b). (ii) The following data gives the number of aircraft accidents that occurred during the various days of a week. Find whether the accidents are uniformly distributed over the week.

Days	Sun	Mon	Tue	Wed	Thu	Fri	Sat
No. of Accidents	14	16	8	12	11	9	14

Solution:

Null Hypothesis (H_0): The accidents are uniformly distributed over the week.

Alternative Null Hypothesis (H_1): The accidents are not uniformly distributed over the week.

The test statistic is given by

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} \quad \chi^2 \text{ distribution with } (n-1) \ d.o.f$$

$$E_i = \frac{total\ no.of\ observations}{n} = \frac{84}{7} = 12 \qquad N = 84, \quad n = 7$$

$$E_i = 12$$

Day	Observed freq	Expected freq	$(O_i - E_i)$	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
Sun	14	12	2	4	0.333333
Mon	16	12	4	16	1.333333
Tue	8	12	-4	16	1.333333
Wed	12	12	0	0	0
Thu	11	12	-1	1	0.083333
Fri	9	12	-3	9	0.75
Sat	14	12	2	4	0.333333
	84	84			4.166

$$\boldsymbol{\chi^2 = 4.166}$$

Table value of $\chi^2_{0.05}$ with $n - 1 = 7 - 1 = 6$ d.o.f is 12.59.

Conclusion:

Since $\chi^2 < \chi^2_{0.05}$, we accept null hypothesis. That is the air accidents are uniformly distributed over the week.

12 (a). Carry out the ANOVA (Analysis of Variance) for the following:

		A	B	C	D
	1	44	38	47	36
	2	46	40	52	43
Workers	**3**	34	36	44	32
	4	43	38	46	33
	5	38	42	49	39

Solution: Let us take the null hypothesis that

1. The 5 workers do not differ with respect to mean productivity

$$i.e., \qquad H_{01} : \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$$

2. The mean productivity is the same for the four different machines.

$$i.e., \quad H_{02}: \mu_1 = \mu_2 = \mu_3 = \mu_4$$

To simplify calculation let us subtract 40 from each value, the new values are

	Machine Type					
		A	B	C	D	Total
Workers	1	4	-2	7	-4	5
	2	6	0	12	3	21
	3	-6	-4	4	-8	-14
	4	3	-2	6	-7	0
	5	-2	2	9	-1	8
	Total	5	-6	38	-17	20

$$Correction\ factor = C.F = \frac{G^2}{N} = \frac{(20)^2}{20} = 20$$

$$SST = Total\ sum\ of\ squares = \sum_i \sum_j y_{ij}^2 - C.F$$

$$= [(4)^2 + (-2)^2 + (7)^2 + (-4)^2 + (6)^2 + (0)^2 + (12)^2 + (3)^2 + (-6)^2 + (-4)^2 + (4)^2 + (-8)^2 + (3)^2 + (-2)^2 + (6)^2 + (-7)^2 + (-2)^2 + (2)^2 + (9)^2 + (-1)^2] - 20$$

$$= 574$$

Between column sum of squares

$$SSC + \frac{(21)^2}{4} + \frac{(-14)^2}{4} + \frac{(0)^2}{4} + \frac{(8)^2}{4} - 20 = 181.5 - 20 = 161.5$$

Between Row sum of squares

$$SSR + \frac{(-6)^2}{5} + \frac{(38)^2}{5} + \frac{(-17)^2}{5} - 20 = 358.8 - 20 = 338.8$$

Error sum of squares

$$SSC = SST - SSC - SSR = 574 - 161.5 - 338.8 = 73.7$$

ANOVA table for two-way classification					
Source of variation	Degrees of freedom	Sum of squares (SS)	Mean sum of squares (MS)	F-Ratio	Table Value
B/w Column	$5-1=4$	$SSC = 161.5$	$MSC = \frac{161.5}{4} = 40.375$	$F_1 = \frac{40.375}{6.14} = 6.576$	$F_{0.05}(4,12) = 3.26$
B/w Row	$4-1=3$	$SSR = 338.8$	$MSR = \frac{338.8}{3} = 112.93$	$F_2 = \frac{112.93}{6.14} = 18.393$	$F_{0.05}(4,12) = 3.26$
Error	$4*3=12$	$SSE = 73.7$	$MSR \frac{73.7}{12} = 6.14$		

Conclusion:

1. $F_1 > F_{0.05}(4,12)$. Hence H_{01} is accepted. That is the 5 workers differ respect to mean productivity.

2. $F_2 > F_{0.05}(3,12)$. Hence H_{02} is rejected. That is the mean productivity is not the same for the four machines.

12. (b) Perform Latin square experiment for the following:

Roam I II III → Three equally spaced concentrations of poison as extracted from the scorpion

Arabic 1 2 3 → Three equally spaced body weights for the animals tested.

Latin A B C → Three equally spaced times of storage of the poison before it is administered to the animals.

	I	II	III
1	0.194 A	0.73 B	1.187 C
2	0.758 C	0.311 A	0.589 B
3	0.369 B	0.558 C	0.311 A

Solution:

Null hypothesis: There is no significant difference between rows, columns and between the treatments.

	1	2	3	**Total**
1	0.194	0.73	1.187	1.485
2	0.758	0.311	0.589	0.916
3	0.369	0.558	0.311	0.511
Total	1.321	1.599	2.087	5.007

$$G = 5.007 \quad and \quad N = 9$$

$$Correction\ factor = \frac{G^2}{N} = \frac{(5.007)^2}{9} = 2.79$$

$$Total\ sum\ of\ squares == \begin{bmatrix} 0.194^2 + 0.73^2 + 1.187^2 + 0.758^2 + 0.311^2 \\ +0.589^2 + 0.369^2 + 0.558^2 + 0.311^2 \end{bmatrix} - C.F$$

$$SST = 3.542 - 2.79 = 0.752$$

Between Column sum of squares

$$SSC = \left[\frac{(\sum x_1)^2}{n} + \frac{(\sum x_2)^2}{n} + \frac{(\sum x_3)^2}{n}\right] - C.F$$

$$SSC = \left[\frac{(1.321)^2}{3} + \frac{(1.599)^2}{3} + \frac{(2.087)^2}{3}\right] - 2.79 = 2.866 - 2.79$$

$$SSC = 0.096$$

Between Row sum of squares

$$SSR = \left[\frac{(\sum y_1)^2}{n} + \frac{(\sum y_2)^2}{n} + \frac{(\sum y_3)^2}{n}\right] - C.F$$

$$SSR = \left[\frac{(2.111)^2}{3} + \frac{(1.658)^2}{3} + \frac{(1.238)^2}{3}\right] - 2.79 = 2.912 - 2.79$$

$$SSR = 0.122$$

Treatment sum of squares SSK

Treatment total $A = 0.194 + 0.311 + 0.311 = 0.816$

$$B = 0.73 + 0.589 + 0.369 = 1.688$$

$$C = 1.187 + 0.758 + 0.558 = 2.503$$

$$SSK = \left[\frac{A^2}{n} + \frac{B^2}{n} + \frac{C^2}{n}\right] - C.F = \left[\frac{(0.816)^2}{3} + \frac{(1.688)^2}{3} + \frac{(2.503)^2}{3}\right] - 2.79$$

$$SSK = 0.47$$

Error sum of squares

$$SSE = SST - SSR - SSC = 0.752 - 0.122 - 0.096 + 0.47 = 0.064$$

$$SSE = 0.064$$

Degrees of freedom: $v_1 = v_2 = v_3 = n - 1 = 3 - 1 = 2,\ \ v_4 = (n-1)\,(n-2) = 2$

ANOVA table for three-way classification					Table Value
Source of variation	D.o.f	Sum of squares (SS)	Mean sum of squares (MS)	F-Ratio	
B/W Column	2	SSC=0.096	$MSC = \frac{SSC}{v_1} = 0.048$	$F_1 = \frac{MSE}{MSC} = 1.5$	$F(2,2) = 19.0$
B/W Row	2	SSR=0.122	$MSR = \frac{SSR}{v_2} = 0.061$	$F_2 = \frac{MSR}{MSE} = 1.906$	$F(2,2) = 19.0$
B/w Treatment	2	SSK=0.47	$MSK = \frac{SSE}{v_3} = 0.235$	$F_3 = \frac{MSK}{MSE} = 7.34$	$F(2,2) = 19.0$
Error	2	SSE=0.064	$MSE = \frac{SSE}{v_3} = 0.032$		

Conclusion:

Accept H_0. That is there is no difference between Row, Column and Treatments.

13. (a). (i). Find the inverse of $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 0 \\ 3 & -1 & -4 \end{bmatrix}$

Solution:

While we find the inverse of the matrix A, the diagonal elements should not be zero. If it is zero, then we rearrange the given matrix in correct form.

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & -1 & -4 \end{bmatrix}$$

We know that $[A, I] = [I, A^{-1}]$

Now, $[A, I] = \begin{bmatrix} 1 & 2 & 0 & \vdots & 1 & 0 & 0 \\ 0 & 1 & 1 & \vdots & 0 & 1 & 0 \\ 3 & -1 & -4 & \vdots & 0 & 0 & 1 \end{bmatrix}$

Now, we need to make $[A.I]$ as a diagonal matrix.

Fix the first row, change second and third row by using first row.

$$[A,I]\sim\left[\begin{array}{ccc:ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & -7 & -4 & -3 & 0 & 1 \end{array}\right] \quad R_3 \Leftrightarrow R_3 - 3R_1$$

Fix the first row & second row, change third row by using second row.

$$[A,I]\sim\left[\begin{array}{ccc:ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 3 & -3 & 7 & 1 \end{array}\right] \quad R_3 \Leftrightarrow R_3 + 7R_2$$

Fix the third row, change first and second row by using third row.

$$[A,I]\sim\left[\begin{array}{ccc:ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & -3 & 0 & -3 & 4 & 1 \\ 0 & 0 & 3 & -3 & 7 & 1 \end{array}\right] \quad R_2 \Leftrightarrow R_3 - 3R_2$$

Fix the second & third row, change first by using second row.

$$[A,I]\sim\left[\begin{array}{ccc:ccc} -3 & 0 & 0 & 3 & -8 & -2 \\ 0 & -3 & 0 & -3 & 4 & 1 \\ 0 & 0 & 3 & -3 & 7 & 1 \end{array}\right] \quad R_1 \Leftrightarrow -3\,R_1 - 2R_2$$

$$[A,I]\sim\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & \frac{3}{-3} & -\frac{8}{-3} & -\frac{2}{-3} \\ 0 & 1 & 0 & -\frac{3}{-3} & \frac{4}{-3} & \frac{1}{-3} \\ 0 & 0 & 1 & -\frac{3}{3} & \frac{7}{3} & \frac{1}{3} \end{array}\right] \quad \begin{array}{c} R_1 \Leftrightarrow R_1/-3 \\ R_2 \Leftrightarrow R_2/-3 \\ R_3 \Leftrightarrow R_3/3 \end{array}$$

$$A^{-1} = \begin{bmatrix} -1 & 8/3 & 2/3 \\ 1 & -4/3 & -1/3 \\ -1 & 7/3 & 1/3 \end{bmatrix}$$

13. (a). (ii). Solve by Gauss Siedal method

$6x + 3y + 12z = 35,\ 8x - 3y + 2z = 20,\ \ 4x + 11y - z = 33$

Solution:

$$6x + 3y + 12z = 35$$
$$8x - 3y + 2z = 20$$
$$4x + 11y - z = 33$$

Since the diagonal elements are not dominant in the coefficient matrix, we rewrite the given equation as follows as follows

$$8x - 3y + 2z = 20$$
$$4x + 11y - z = 33$$
$$6x + 3y + 12z = 35$$

From the above equation, we have

$$x = \frac{1}{8}(20 + 3y - 2z)$$

$$y = \frac{1}{11}(33 - 4x + z)$$

$$z = \frac{1}{12}(35 - 6x - 3y)$$

Gauss Siedal Method:

We form the Iterations in the table

Iteration	x	y	z

1	2.5	2.09	1.14
2	2.99	2.05	0.909
3	3.04	1.98	0.90
4	3.02	1.98	0.91
5	3.02	1.98	0.91
6	3.02	1.98	0.91

Hence the solution is $\boldsymbol{x = 3.02,\ \ y = 1.98\ \ and\ \ z = 0.91}$.

13. (b). (i) Using Gauss Jordan method, solve the following system

$10x + y + z = 12,\ \ 2x + 10y + z = 13,\ \ \ x + y + 5z = 7$.

Solution: Let the given system of equations be

$$10x + y + z = 12$$
$$2x + 10y + z = 13$$
$$x + y + 5z = 7$$

The given system is equivalent to $A\,X = B$

$$\begin{bmatrix} 10 & 1 & 1 \\ 2 & 10 & 1 \\ 1 & 1 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 13 \\ 7 \end{bmatrix}$$

$Here\ \ [A, B] = [\quad]$

Now, we need to make A as a diagonal matrix.

$Fix\ the\ first\ \ row, change\ second\ \ and\ third\ row\ by\ using\ first\ row.$

$$[A, B] \sim \begin{bmatrix} 10 & 1 & 1 & 12 \\ 0 & 98 & 8 & 106 \\ 0 & 9 & 49 & 58 \end{bmatrix} \quad \begin{matrix} R_2 \Leftrightarrow 10R_2 - 2\,R_1 \\ R_3 \Leftrightarrow 10R_3 - R_1 \end{matrix}$$

$Fix\ the\ first\ \&\ second\ \ row, change\ the\ \ third\ row\ by\ using\ second\ \ row.$

$$[A, B] \sim \begin{bmatrix} 10 & 1 & 1 & 12 \\ 0 & 98 & 8 & 106 \\ 0 & 0 & 4730 & 4730 \end{bmatrix} \quad R_3 \Leftrightarrow 98R_3 - 9\,R_2$$

$$[A, B] \sim \begin{bmatrix} 10 & 1 & 1 & 12 \\ 0 & 98 & 8 & 106 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad R_3 \Leftrightarrow R_3/4730$$

$Fix\ the\ third\ \ row, change\ first\ \ and\ second\ \ row\ by\ using\ third\ row.$

$$[A, B] \sim \begin{bmatrix} -10 & -1 & 0 & -11 \\ 0 & 98 & 0 & 98 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad \begin{matrix} R_1 \Leftrightarrow R_1 - R_3 \\ R_2 \Leftrightarrow R_2 - 8\,R_3 \end{matrix}$$

$$[A, B] \sim \begin{bmatrix} -10 & -1 & 0 & -11 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad R_2 \Leftrightarrow R_2/98$$

$Fix\ the\ second\ \ \&\ \ third\ \ row, change\ first\ \ by\ using\ second\ row.$

$$[A, B] \sim \begin{bmatrix} -10 & 0 & 0 & -10 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad R_1 \Leftrightarrow R_1 + R_2$$

Which is a diagonal matrix, from the matrix, we have

$$\Longrightarrow\ x = 1,\ \ y = 1,\ \ z = 1$$

13. (b) (ii). Find all the Eigen value of $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ using power method.

Using $x_1 = [1 \quad 0 \quad 0]^T$ as initial vector.

Solution : Let $X_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ be the initial vector.

Therefore,

$$A\,X_1 = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = 1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = 1\,X_2$$

$$A\,X_2 = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 0 \end{bmatrix} = 7 \begin{bmatrix} 1 \\ 0.4286 \\ 0 \end{bmatrix} = 7\,X_3$$

$$A\,X_3 = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0.4286 \\ 0 \end{bmatrix} = \begin{bmatrix} 3.574 \\ 1.8572 \\ 0 \end{bmatrix} = 3.574 \begin{bmatrix} 1 \\ 0.52 \\ 0 \end{bmatrix} = 3.574\,X_4$$

$$A\,X_4 = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0.52 \\ 0 \end{bmatrix} = \begin{bmatrix} 4.12 \\ 2.04 \\ 0 \end{bmatrix} = 4.12 \begin{bmatrix} 1 \\ 0.4951 \\ 0 \end{bmatrix} = 4.12\,X_5$$

$$A\,X_5 = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0.4951 \\ 0 \end{bmatrix} = \begin{bmatrix} 3.9706 \\ 1.9902 \\ 0 \end{bmatrix} = 3.9706 \begin{bmatrix} 1 \\ 0.5012 \\ 0 \end{bmatrix} = 3.9706\,X_6$$

$$A\,X_6 = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0.5012 \\ 0 \end{bmatrix} = \begin{bmatrix} 4.0072 \\ 2.0024 \\ 0 \end{bmatrix} = 4.0072 \begin{bmatrix} 1 \\ 0.4997 \\ 0 \end{bmatrix} = 4.0072\,X_7$$

$$A\,X_7 = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0.4997 \\ 0 \end{bmatrix} = \begin{bmatrix} 3.9982 \\ 1.9994 \\ 0 \end{bmatrix} = 3.9982 \begin{bmatrix} 1 \\ 0.5000 \\ 0 \end{bmatrix} = 3.9982\,X_8$$

$$A\,X_8 = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0.50 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix} = 4 \begin{bmatrix} 1 \\ 0.5000 \\ 0 \end{bmatrix} = 4\,X_9$$

∴ The dominant **Eigen value** = 4.

Corresponding **Eigen vector** is $\begin{bmatrix} 1 \\ 0.5 \\ 0 \end{bmatrix}$.

14. (a). (i). Taking $h = \pi/10$, evaluate $\int_0^{\pi} \sin x \; dx$ by Simpson's $\frac{1}{3}$ rule. Verify the answer with integration.

Solution:

Here $y(x) = \sin x$, $h = \frac{\pi}{10}$. We form a table

$x:$	0	$\frac{\pi}{10}$	$\frac{2\pi}{10}$	$\frac{3\pi}{10}$	$\frac{4\pi}{10}$	$\frac{5\pi}{10}$	$\frac{6\pi}{10}$	$\frac{7\pi}{10}$	$\frac{8\pi}{10}$	$\frac{9\pi}{10}$	π
$y:$	0	0.3090	0.5878	0.8090	0.9511	1.0	0.9511	0.8090	0.5878	0.3090	0

Simpson's 1/3 rule :

$$\int_0^{\pi} \sin x \; dx = \left(\frac{h}{3}\right) [(y_0 + y_{10}) + 4(y_1 + y_3 + y_5 + y_7 + y_9) + 2(y_2 + y_4 + y_6 + y_8)]$$

$$= \frac{\left(\frac{\pi}{10}\right)}{3} [(0+0) + 2(0.5878 + 0.9511 + 0.9511 + 0.5878) + 4(0.3090 + 0.8090 + 1 + 0.8090 + 0.3090)]$$

$$= \frac{\pi}{30}\,[19.0996]$$

$$\int_0^{\pi} \sin x \; dx = 2.0091.$$

By Actual Integration:

$$\int_0^{\pi} \sin x \; dx = [-\cos x]_0^{\pi} = -[\cos \pi - \cos 0] = -[-1-1] = 2.$$

14. (a). (ii). Use Lagrange's formula to fit a polynomial to the following data hence find $y\ (x = 1)$.

x:	-1	0	2	3
y:	-8	3	1	12

Solution:

x :	$-1\ x_0$	$0\ x_1$	$2\ x_2$	$3\ x_3$
y :	$-8\ y_0$	$3\ y_1$	$1\ y_2$	$12\ y_3$

Lagrange's interpolation formula, we have

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)}\,y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)}\,y_1$$

$$+\frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)}\,y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)}\,y_3$$

$$f(x) = \frac{(x-0)(x-2)(x-3)}{(-1-0)(-1-2)(-1-3)}\,(-8) + \frac{(x+1)(x-2)(x-3)}{(0+1)(0-2)(0-3)}\,(3) + \frac{(x+1)(x-0)(x-3)}{(2+1)(2-0)(2-3)}\,1$$
$$+\frac{(x+1)(x-0)(x-2)}{(3+1)(3-0)(3-2)}\,(12)$$

$$f(x) = \frac{x(x-2)(x-3)}{-12}(-8) + \frac{(x+1)(x-2)(x-3)}{6}(3) + \frac{x(x+1)(x-3)}{-6} + \frac{x(x+1)(x-2)}{12}\ (12)$$

$$= \left(\frac{2}{3}\right)[(x^2-2x)(x-3)] + \left(\frac{1}{2}\right)[(x^2-x-2)(x-3)] - \left(\frac{1}{6}\right)[(x^2+x)(x-3)] + [(x^2+x)(x-2)]$$

$$= \left(\frac{2}{3}\right)[x^3-5x^2+6x] + \left(\frac{1}{2}\right)[x^3-4x^2+x+6] - \left(\frac{1}{6}\right)[x^3-2x^2-3x] + [x^3-x^2-2x]$$

$$= x^3\left[\frac{2}{3}+\frac{1}{2}-\frac{1}{6}+1\right] + x^2\left[\frac{2}{3}(-5)+\frac{1}{2}(-4)-\frac{1}{6}(-2)-1\right] + x\left[\frac{2}{3}(6)+\frac{1}{2}-\frac{1}{6}(3)-2\right] + \left[\frac{1}{6}(6)\right]$$

$$= x^3\left[\frac{2}{3}+\frac{1}{2}-\frac{1}{6}+1\right] + x^2\left[\frac{2}{3}(-5)+\frac{1}{2}(-4)-\frac{1}{6}(-2)-1\right] + x\left[\frac{2}{3}(6)+\frac{1}{2}-\frac{1}{6}(3)-2\right] + \left[\frac{1}{6}(6)\right]$$

$$f(x) = 2x^3 - 6x^2 + 2x + 1$$

14. (b) (ii). Evaluate $\int_0^6 \frac{1}{1+x^2}\,dx$ using Trapezoidal rule. Verify the answer with direct integration.

Solution:

Here $y(x) = \frac{1}{1+x^2}$. Range $= b - a = 6 - 0 = 6$

So we divide 6 equal intervals with $h = \frac{6}{6} = 1$.

We form a table

x :	0	1	2	3	4	5	6

$y = \frac{1}{1+x^2}$:	1	0.500	0.200	0.100	0.058824	0.038462	0.27027

Trapezoidal rule

$$\int_0^6 \frac{1}{1+x^2}\,dx = \frac{h}{2}\,[(y_0 + y_6) + 2(y_1 + y_2 + y_3 + y_4 + y_5)]$$

$$= \frac{1}{2}\,[(1 + 0.027027) + 2(0.5 + 0.2 + 0.1 + 0.058824 + 0.038462)]$$

$$= \frac{1}{2}\,[2.821599]$$

$$\int_0^6 \frac{1}{1+x^2}\,dx = 1.4107995.$$

By Actual Integration:

$$\int_0^6 \frac{1}{1+x^2}\,dx = [\tan^{-1} x]_0^6 = \tan^{-1} 6 - \tan^{-1} 0 = 1.40564765.$$

14. (b). (ii) Find $y(1976)$ from the following

X:	1941	1951	1961	1971	1981	1991
Y:	20	24	29	36	46	51

Solution: We form the difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1941	20				
		4			
1951	24		1		
		5		1	
1961	29		2		0
		7		1	
1971	36		3		**1**
		10		**2**	
1981	46		**5**		
		5			
1991	**51**				

The Newton's backward formula is

The Newton's backward interpolation formula is

$$y\,(x) = y\,(x_n + ph) \;= y_n + \frac{V}{1\,!}\,\nabla\, y_n + \frac{V(V+1)}{2\,!}\,\nabla^2 y_n + \frac{V(V+1)(V+2)}{3\,!}\,\nabla^3 y_n + \cdots \ldots\ldots$$

$where \;\; u = \frac{x - x_n}{h}$. Here $x_n = 1991$ & $h = 10$, $x = 1976$

$Let \;\; u = \frac{x - x_n}{h} = \frac{1976-1991}{10} = -1.5$

$$y\,(1976) = 51 + \frac{(-1.5)}{1\,!}\,(5) + \frac{(-1.5)[-1.5+1]}{2\,!}\,(5) + \frac{(-1.5)[(-1.5)+1][-1.5+2]}{3\,!}\,(2)$$

$= 51 - 7.5 + 1.875 + 0.125$

$\boldsymbol{y\,(1976) = 45.5}$

15. (a) (i). Use Modified Euler's method, with $h = 0.1$ to find the solution of $y' = x^2 + y^2$ with $y(0) = 0$ in $0 \le x \le 5$.

Solution:

Given $y' = x^2 + y^2,\ \ y(0) = 0,\ \ x_0 = 0,\ \ y_0 = 0,\ \ h = 0.1.$

The Modified Euler's formula is

$$\boldsymbol{y_{n+1}(x_n + h) = y_n + h\, f\left(x_n + \frac{h}{2},\ y_n + \frac{h}{2}\, f(x_n, y_n)\right)\ ,\ n = 0, 1, 2 \ldots}$$

To find $\boldsymbol{y(0.1)}$ **:**

Put $n = 0$, equation (1) becomes

$$y_1(x_0 + h) = y_0 + h\, f\left(x_0 + \frac{h}{2},\ y_0 + \frac{h}{2}\, f(x_0, y_0)\right)$$

$We\ have\ \ x_0 = 0,\ \ y_0 = 1\ ,\ \ h = 0.1\ \ \&\ \ f(x, y) = x^2 + y^2$

$$\therefore\quad y_1(0 + 0.1) = 1 + (0.1)\ f\left(0 + \frac{0.1}{2},\ 1 + \frac{0.1}{2}\, f(0, 1)\right)$$

$$y_1(0.1) = 1 + (0.1)\, f(0.05,\ 1 + 0.05\,[0^2 + 1^2])$$

$$= 1 + (0.1)\, f(0.05,\ 1 + 0.05\,[1])$$

$$= 1 + (0.1)\, f(0.05,\ 1.05)$$

$$= 1 + (0.1)\,[(0.05)^2 + (1.05)^2]$$

$$= 1 + (0.1)\,[1.105]$$

$$= 1 + 0.1105$$

$$y_1(0.1) = 1.1105$$

$\boldsymbol{y_1(0.1) = 1.1105}\qquad [\boldsymbol{y(x_1) = y_1}]\ \ \Rightarrow\ x_1 = \boldsymbol{0.1}\ \ \&\ \ \boldsymbol{y_1 = 1.1105}$

15. (a). (ii). Using Milne's method, obtain the solution of $\frac{dy}{dx} = x - y^2\ at\ x = 0.8,$ given

$y(0) = 0,\ y(0.2) = 0.02,\ \ y(0.4) = 0.0795,\ \ y(0.6) = 0.1762.$

Solution: Given $y' = x - y^2$

$y(0) = 1$	$y(x_0) = y_0$	$x_0 = 0$	$y_0 = 0$
$y(0.2) = 0.02$	$y(x_1) = y_1$	$x_1 = 0.2$	$y_1 = 0.02$

$y(0.4) = 0.0795$	$y(x_2) = y_2$	$x_2 = 0.4$	$y_2 = 0.0795$
$y(0.6) = 0.1762$	$y(x_3) = y_3$	$x_3 = 0.6$	$y_3 = 0.1762$

$\boldsymbol{Here \quad h = 0.2}$ **and** $\boldsymbol{n = 3 \quad [Highest\ value\ of\ \ x\ \ is\ \ x_3. \quad \therefore n = 3]}$

The Milne's Predictor formula is

$$y_{n+1,P}(x_n + h) = y_{n-3} + \frac{4h}{3}[2\ y'_{n-2} - y'_{n-1} + 2\ y'_n] \quad \dots\dots(1)$$

To Find y(0.8) :

Put n=3 in equation (1), we have

$$y_{4,P}(x_3 + h) = y_0 + \frac{4h}{3}[2\ y'_1 - y'_2 + 2\ y'_3] \quad \dots\dots(2)$$

$Given \quad y' = x - y^2$

$x_1 = 0.2$	$y_1 = 0.02$	$y'_1 = x_1 - y_1^2$	$y'_1 = (0.2) - (0.02)^2$	$y'_1 = 0.1996$
$x_2 = 0.4$	$y_2 = 0.0795$	$y'_2 = x_2 - y_2^2$	$y'_2 = (0.4) - (0.0795)^2$	$y'_2 = 0.3937$
$x_3 = 0.6$	$y_3 = 0.1762$	$y'_3 = x_3 - y_3^2$	$y'_3 = (0.6) - (0.1762)^2$	$y'_3 = 0.5690$

Equation (2) becomes

$$y_{4,P}(x_3 + h) = y_0 + \frac{4h}{3}[2\ y'_1 - y'_2 + 2\ y'_3]$$

$$y_{4,P}(0.6 + 0.2) = 0 + \frac{4(0.2)}{3}[2\ (0.1996) - (0.3937) + 2\ (0.5690)]$$

$$y_{4,P}(0.8) = \frac{0.8}{3}[1.1435]$$

$$\boldsymbol{y_{4,P}(0.8) = 0.3049 \quad [y(x_4) = y_4, \quad x_4 = 0.8 \quad \& \quad y_4 = 0.3049\ ]}$$

The Milne's Corrector formula is

$$y_{n+1,C}(x_n + h) = y_{n-1} + \frac{h}{3}[\ y'_{n-1} + 4y'_n + \ y'_{n+1}] \quad \dots\dots(3)$$

Put n=3 in equation (3), we have

$$y_{4,C}(x_3 + h) = y_2 + \frac{h}{3}[\ y'_2 + 4y'_3 + \ y'_4] \quad \dots\dots(3)$$

$x_4 = 0.8$	$y_4 = 0.3049$	$y'_4 = x_4 - y_4^2$	$y'_4 = \ 0.8 - (0.3049)^2$	$y'_4 = 0.707$

Equation (4) becomes

$$y_{4,C}(0.6 + 0.2) = 0.0795 + \frac{(0.2)}{3}[0.3937 + 4(0.5690) + 0.707]$$

$$y_{4,C}(0.8) = 0.07957 + \frac{(0.2)}{3}[3.376]$$

$$\boldsymbol{y_{4,C}(0.8) = 0.3046 \quad [y(x_4) = y_4, \quad x_4 = 0.8 \ \& \ y_4 = 0.3046\ ]}$$

To Find (1.0) :

Put n=3 in equation (1), we have

$$y_{4,P}(x_3+h) = y_0 + \frac{4h}{3}[2\, y_1' - y_2' + 2\, y_3'] \quad \dots\dots (2)$$

$Given \quad y' = x - y^2$

$x_1 = 0.2$	$y_1 = 0.02$	$y_1' = x_1 - y_1^2$	$y_1' = (0.2) - (0.02)^2$	$y_1' = 0.1996$
$x_2 = 0.4$	$y_2 = 0.0795$	$y_2' = x_2 - y_2^2$	$y_2' = (0.4) - (0.0795)^2$	$y_2' = 0.3937$
$x_3 = 0.6$	$y_3 = 0.1762$	$y_3' = x_3 - y_3^2$	$y_3' = (0.6) - (0.1762)^2$	$y_3' = 0.5690$

Equation (2) becomes

$$y_{4,P}(x_3+h) = y_0 + \frac{4h}{3}[2\, y_1' - y_2' + 2\, y_3']$$

$$y_{4,P}(0.6+0.2) = 0 + \frac{4(0.2)}{3}[2\,(0.1996) - (0.3937) + 2\,(0.5690)]$$

$$y_{4,P}(0.8) = \frac{0.8}{3}[1.1435]$$

$$\boldsymbol{y_{4,P}(0.8) = 0.3049} \quad [\boldsymbol{y(x_4) = y_4,\quad x_4 = 0.8 \;\; \& \;\; y_4 = 0.3049}]$$

The Milne's Corrector formula is

$$y_{n+1,C}(x_n+h) = y_{n-1} + \frac{h}{3}[\, y_{n-1}' + 4y_n' + y_{n+1}'] \quad \dots\dots (3)$$

Put n=3 in equation (3), we have

$$y_{4,C}(x_3+h) = y_2 + \frac{h}{3}[\, y_2' + 4y_3' + y_4'] \quad \dots\dots (3)$$

$x_4 = 0.8$	$y_4 = 0.3049$	$y_4' = x_4 - y_4^2$	$y_4' = 0.8 - (0.3049)^2$	$y_4' = 0.707$

Equation (4) becomes

$$y_{4,C}(0.6+0.2) = 0.0795 + \frac{(0.2)}{3}[0.3937 + 4(0.5690) + 0.707]$$

$$y_{4,C}(0.8) = 0.07957 + \frac{(0.2)}{3}[3.376]$$

$$\boldsymbol{y_{4,C}(0.8) = 0.3046} \quad [\boldsymbol{y(x_4) = y_4,\quad x_4 = 0.8 \;\; \& \;\; y_4 = 0.3046}]$$

Result:

$y_{4,P}(0.8) = 0.3049 \quad \& \quad y_{4,C}(0.8) = 0.3046$

15. (b). (i). Use R.K method fourth order to the $y(0.2)$ if $\frac{dy}{dx} = x + y^2,\; y(0) = 1, h = 0.1.$

Solution:

$Given \quad \frac{dy}{dx} = x^2 + y^2 = f(x,y)\;,\; y(0) = 1 \qquad \Rightarrow x_0 = 0,\; y_0 = 1\;,\; h = 0.1$

The algorithm for fourth order R-K method is

To find $y(0.1)$: $[x_0 = 0.0,\ \ y_0 = 1,\ \ h = 0.1,\ f(x,y) = x^2 + y^2]$

$$k_1 = h\,f(x_0, y_0) = 0.1\ f(0,\ \ 1\ ) = 0.1\ [(0)^2 + (1)^2] = 0.1[1]$$

$\boldsymbol{k_1 = 0.1}$

$$k_2 = h\,f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = 0.1\ f\left(0 + \frac{0.1}{2},\ \ 1 + \frac{0.1}{2}\ \right) = 0.1\ f(0.05,\ \ 1.05)$$

$$k_2 = 0.1\ [(0.05)^2 + (1.05)^2] = 0.11525$$

$\boldsymbol{k_2 = 0.11525}$

$$k_3 = h\,f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right) = 0.1\ f\left(0 + \frac{0.1}{2}, 1 + \frac{0.111525}{2}\right) = 0.2f(0.05,\ \ 1.057625)$$

$$k_3 = 0.1\ [(0.05)^2 + (1.057625)^2] = 0.116857$$

$\boldsymbol{k_3 = 0.116857}$

$$k_4 = h\,f(x_0 + h, y_0 + k_3) = 0.1\ \ f(0 + 0.1,\ \ 1 + 0.116857) = 0.1f(0.1,\ \ 1.116857)$$

$$k_4 = 0.1\ [(0.1)^2 + (1.116857)^2] = 0.1347$$

$\boldsymbol{k_4 = 0.1347}$

$$\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = \frac{1}{6}(0.1 + 2(0.11525) + 2(0.116857) + 0.1347)\ \ \boldsymbol{\Delta y = 0.11649}$$

$$y(x_0 + h) = y(x_0) + \Delta y = y_0 + \Delta y \quad \Rightarrow \ y(0.0 + 0.2) = 1 + 0.11649 = 1.11649$$

$\boldsymbol{y(0.1) = 1.1165} \quad \boldsymbol{[y(x_1) = y_1]}$

To find $y(0.2)$: $\boldsymbol{[x_1 = 0.1,\ \ y_1 = 1.1165,\ \ h = 0.1,\ f(x,y) = x^2 + y^2]}$

$$k_1 = h\,f(x_1, y_1) = 0.1\ f(0.1,\ \ 1.1165\ ) = 0.1\ [(0.1)^2 + (1.1165)^2]$$

$\boldsymbol{k_1 = 0.1347}$

$$k_2 = h\,f\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right) = 0.1\ f\left(0.1 + \frac{0.1}{2},\ \ 1.1165 + \frac{0.1347}{2}\ \right) = 0.1\ f(0.15,\ \ 1.18385)$$

$$k_2 = 0.1\ [(0.15)^2 + (1.18385)^2] = 0.1552$$

$\boldsymbol{k_2 = 0.1552}$

$$k_3 = h\,f\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right) = 0.1\ f\left(0.1 + \frac{0.1}{2}, 0.1165 + \frac{0.1552}{2}\right) = 0.1f(0.15,\ \ 1.1941)$$

$$k_3 = 0.1\ [(0.15)^2 + (1.1941)^2] = 0.1576$$

$\boldsymbol{k_3 = 0.1576}$

$$k_4 = h\,f(x_1 + h, y_1 + k_3) = 0.1\ \ f(0.1 + 0.1,\ \ 1.1165 + 0.1576) = 0.1\ f(0.2,\ \ 1.2741)$$

$$k_4 = 0.2\ [(0.2)^2 + (1.2741)^2] = 0.1823$$

$\boldsymbol{k_4 = 0.1843}$

$$\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = \frac{1}{6}(0.1347 + 2(0.1552) + 2(0.1576) + 0.1843)\ \ \boldsymbol{\Delta y = 0.1571}$$

$$y(x_1 + h) = y(x_1) + \Delta y \quad \Rightarrow \ y(0.1 + 0.1) = y(0.1) + 0.1571 = 1.1165 + 0.1571$$

$\boldsymbol{y(0.2) = 1.2736}$

15. (b). (ii) Solve $\boldsymbol{u_{n+2} - 4u_{n+1} + 4u_n = 2^n}$.

Solution:

The given difference equation is

$$\boldsymbol{[E^2 - 4E + 4]y_n = 2^n}$$

The auxiliary equation is $\boldsymbol{m^2 - 4m + 4 = 0}$

$$i.e., \quad (m-2)^2 = 0$$

$$i.e., \quad m = 2, \ 2$$

$$C.F = (Ax + B)2^n$$

$$Particulur\ Integral\ \ P.I = \frac{1}{(E-2)^2}\ 2^n$$

$$= \frac{n(n-1)}{2!}.\,2^{n-2}$$

$$= n(n-1)2^{n-3}$$

$U = (Ax + B)2^n + n(n-1)2^{n-3}$s