STATISTICS AND NUMERICAL METHODS

QUESTION III APRIL / MAY 2011

1. What are the applications of t distributions?

* Test the hypothesis about the population mean for small samples

* Test the hypothesis about the difference between two means for small samples.

2. Define Type I and Type II errors in taking a decision.

Solution:

(i). Type-I error : Reject H_0 when it is true.

(ii). Type-II error : Accept H_0 when it is wrong.

3. What are the conditions for the validity of ψ^2 test?

Solution:

1. The experimental data (samples) must be independent to each other.
2. The total frequency (no. of observations in the sample) must be large, say ≥ 50.
3. All the individual data's should be greater than 5.
4. The no. of classes n must lies in $4 \leq n \leq 16$.

4. Write any two differences between RBD and CRD.

Solution:

i. This design is more efficient than CRD. That is it has less experimental error.

ii. The statistical analysis for this design is simple and rapid.

5. What is the order of convergence and also state the error term for Newton Raphson method?

Solution:

The Criterion for convergence of Newton- Raphson -method is

$$|f(x)\ f''(x)| < |\ f'(x)|^2 \quad \textit{in the interval considered.}$$

The order of convergence of Newton- Raphson –method is 2.

6. Find the dominant Eigen value of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ by power method.

Solution:

Solution : Let $X_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ be the initial vector.

Therefore,

$$A\,X_1 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} = 3 \begin{bmatrix} 0.33 \\ 1 \end{bmatrix} = 3\,X_2$$

$$A\,X_2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 0.33 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.33 \\ 4.99 \end{bmatrix} = 4.99 \begin{bmatrix} 0.47 \\ 1 \end{bmatrix} = 4.99\ X_3$$

$$A\,X_3 = \begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\begin{bmatrix}0.47\\ 1\end{bmatrix} = \begin{bmatrix}2.47\\ 5.41\end{bmatrix} = 5.41\begin{bmatrix}0.46\\ 1\end{bmatrix} = 5.41\ X_4$$

$$A\,X_4 = \begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\begin{bmatrix}0.46\\ 1\end{bmatrix} = \begin{bmatrix}2.46\\ 5.38\end{bmatrix} = 5.38\begin{bmatrix}0.46\\ 1\end{bmatrix} = \ X_5$$

$$A\,X_5 = \begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\begin{bmatrix}0.46\\ 1\end{bmatrix} = \begin{bmatrix}2.46\\ 5.38\end{bmatrix} = 5.38\begin{bmatrix}0.46\\ 1\end{bmatrix} = \ X_6$$

$\therefore$ The dominant **Eigen value** $=\ 5.38$.

7. Using Trapezoidal rule, evaluate $\int_0^1 \frac{dx}{1+x^2}$ with $h = 0.2$. Hence obtain an approximate value of π.
Solution:

Here $y(x) = \frac{1}{1+x^2}$. We form a table

x :	0	0.2	0.4	0.6	0.8	1.0
$y = \dfrac{1}{1+x^2}$:	1	0.961	0.862	0.735	0.609	0.5

Trapezoidal rule

$$\int_0^1 \frac{1}{1+x^2}\,dx = \frac{1}{2}\ [(1+0.5) + 2(0.961+0.862+0.735+0.609)]$$

$$I = 3.917$$

By Actual Integration:

$$I = \int_0^1 \frac{1}{1+x^2}\,dx = [\tan^{-1} x]_0^1 = \tan^{-1} 1 - \tan^{-1} 0 = \pi$$

Hence $I = 3.197 = \pi$

8. State the formula to find the second order derivative using the forward differences.
Solution:

The second derivative of $y\ at\ x = x_0 + ph\ \ is$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\ \Delta^2 y_0 + (p-1)\ \Delta^3 y_0 + \frac{6p^2 - 18\,p + 11}{12}\ \Delta^4 y_0 + \cdots\right]$$

9. Given $y' = -y$ and $y(0) = 1$, determine the value of $y(0.01)$ by Euler's method.
Solution:

$Given\ \ y' = f(x,y) = -y \quad and\ \ y(0) = 1 \quad \Rightarrow\ x_0 = 0,\ \ y_0 = 1$

To find h : $\quad Since\ \ y(0) = 1 \quad \Rightarrow\ \ y(x_0) = y_0$

$\therefore\ \ h = x_1 - x_0 = 0.01 - 0.0 = 0.01$

The Euler's formula is $\quad \boldsymbol{y_{n+1}(x_n + h) = y_n + h\,[f(x_n, y_n)]\ ,\ n = 0, 1, 2, \ldots}$ **To find**

$$y_1(x_0 + h) = y_0 + h\,[f(x_0, y_0)]$$

$$\therefore\quad y_1(0 + 0.01) = 1 + (0.01)\,[f(0, 1)] = 1 + (0.01)\,[-1] = 1 - 0.01$$

$$\boldsymbol{y_1(0.01) = 0.99}$$

10. Write the Milne's Predictor-corrector formula.

Solution:
The Milne's Predictor formula is

$$y_{n+1,P}(x_n + h) = y_{n-3} + \frac{4h}{3}[2\,y'_{n-2} - y'_{n-1} + 2\,y'_n]$$

The Milne's Corrector formula is

$$y_{n+1,C}(x_n + h) = y_{n-1} + \frac{h}{3}[\,y'_{n-1} + 4y'_n + y'_{n+1}]$$

Part - B

11. (a) (i). In a random sample of 1000 people from city A, 400 are found to be consumers of wheat. In a sample of 800 from city B, 400 are found to be consumers of wheat. Does this data give a significant difference between the two cities as far as the proportion of wheat consumers is concerned?

Solution:

$$Given\ that\quad n_1 = 1000,\quad x_1 = 400 \qquad and \qquad n_2 = 800,\quad x_2 = 400$$

$$p_1 = Sample\ prop\ of\ consume\ alcohol\ in\ A \qquad p_1 = \frac{400}{1000} = 0.4 \quad \& \quad q_1 = 0.6$$

$$p_2 = Sample\ prop\ of\ consume\ alcohol\ in\ B \qquad p_2 = \frac{x_2}{n_2} = \frac{400}{800} = 0.5 \quad \& \quad q_2 = 0.5$$

Null Hypothesis:

$H_0 \ :\ P_1 = P_2$ $\quad i.e.,$ there is no difference between A & B

Alternative hypothesis:

$H_1 \ :\ P_1 \neq P_2$

Here Population Proportion P is not known.

$$\therefore\quad P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

$$P = \frac{1000 * 0.4 + 800 * 0.5}{1000 + 800} = 0.444$$

$$\therefore\quad P = 0.444 \quad and \quad Q = 0.556$$

$$The\ test\ statistic\ is\ given\ by\ \ z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$z = \frac{0.5 - 0.4}{\sqrt{[0.444 * 0.556]\left(\frac{1}{1000} + \frac{1}{800}\right)}} = \frac{0.1}{0.02356}$$

$$z = 4.24 \quad app$$

At 5% level of significance, the table value for $z_{\propto} = 1.96$.

$$|calculated\ value\ | > tabulated\ value$$

$$|4.24| > 1.96 \quad \Rightarrow reject\ \ H_0$$

Conclusion: We reject the null hypothesis. That is there is some difference b/w two groups.

11. (a). (ii). 4 coins were tossed 160 times and the following results were obtained :

No. of heads:	0	1	2	3	4
Observed frequencies:	1	7	52	54	31

Under the assumption that the coins are unbiased, find the expected frequencies of getting $0, 1,2,3, 4$ heads and test the goodness of fit.

Solution:

H_0: The coin is unbiased.

H_1: The coin is biased.

Probability of getting head $p = \frac{1}{2}$.
Probability of getting tail $q = \frac{1}{2}$.
The expected frequencies are $P(x) = nC_x\, p^x\, q^{n-x}, \quad x = 0,1,2..$

$$P(x) = 4C_x\, p^x\, q^{4-x}, \quad x = 0,1,2..$$

$$P(0\ head) = P(x=0) = 4C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{4-0} = 0.0625$$

$$P(1\ head) = P(x=1) = 4C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{4-1} = 0.25$$

$$P(2\ head) = P(x=2) = 4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{4-2} = 0.375$$

$$P(3\ head) = P(x=3) = 4C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{4-3} = 0.25$$

$$P(4\ head) = P(x=4) = 4C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{4-4} = 0.0625$$

No. of Heads	O_i	$E_i = N * P(x)$	$\frac{(O_i - E_i)^2}{E_i}$
0	17	10	4.9
1	52	40	3.6
2	54	60	0.6
3	31	40	2.025
4	6	10	1.6
		N=160	12.725

$$\chi^2 = \mathbf{12.725}$$

At 5% LOS with $n - 1 = 5 - 1 = 4$ d.o.f is 9.488
$|Cacl\ value| > Table\ value$
We reject H_0, The coin is biased one.

OR

11. (b). (i). The heights of 10 males of a given locality are found to be 67,62,68, 61, 68,70, 64, 64,66 inches. Is it reasonable to believe that the average height is greater than 64 inches?

Solution:

x_i	70	67	62	68	61	68	70	64	64	66	$\sum x = 660$
$x_i - \bar{x}$	4	1	-4	2	-5	2	4	-2	-2	0	
$(x_i - \bar{x})^2$	16	1	16	4	25	4	16	4	4	0	$\sum (x_i - \bar{x})^2 = 90$

$$\bar{x} = \frac{\sum x_i}{n} = \frac{660}{10} = 66$$

$$\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} = \frac{90}{10-1} = \frac{90}{9} = 10$$

$$\therefore \quad \sigma = \sqrt{10} = 3.162$$

Hence $n = 10, \ \bar{x} = 66, \quad \sigma = 3.162, \quad \mu = 64$

Null Hypothesis : $H_0 : \mu = 64$ i.e., the average height is equal to 64 inches.

Alternative Hypothesis : $H_1 : \mu > 64$ **(Right tailed test)**

The test statistic is given by

$$t = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \quad with\, n - 1\ degrees\ of\ freedom$$

$$t = \frac{66 - 64}{\frac{3.162}{\sqrt{10}}} = \frac{2}{1} = 2$$

$$|t| = 2$$

The critical value for t for a right tailed test at 5% level of significance with $10 - 1 = 9$ degrees of freedom is 1.833.

Calculated value =2 and Tabulated value =1.833

$|Calculated\ value\ | \leq Tabulated\ value \quad then\ accept\ H_0$

$But \quad |2| > 1.833 \quad Reject\ \ H_0$

Conclusion:

$|t_{\propto}| > t$, we reject H_0. That is the average height is greater than 64 inches.

11. (b). (ii). Test of the fidelity and selectivity of 190 radio receivers produced the results shown in the following table :

		Fidelity	
Selectivity	Low	Average	High
Low	6	12	32
Average	33	61	18
High	12	15	0

Use the 0.ol level of significance to test whether there is a relationship between fidelity and selectivity.

Solution:

$\boldsymbol{H_0}$: There is no relationship between fidelity and selectivity.

$\boldsymbol{H_1}$: There is some relationship between fidelity and selectivity.

The test statistic is given by

$$\chi^2 = \sum_{i=1}^{r}\sum_{j=1}^{s}\frac{\left(O_{ij} - E_{ij}\right)^2}{E_{ij}} \quad \chi^2\ distribution\ with \quad n = (r-1)(s-1) - 1 \quad d.o.f$$

$$E_{ij} = \frac{R_i C_j}{N} \quad ; \quad i = 1,2, \ldots r \quad and \quad j = 1,2, \ldots . s$$

The expected frequencies are

$$E(6) = \frac{52 * 50}{190} = 13.684\ ; \qquad E(12) = \frac{88 * 50}{190} = 23.158$$

$$E(32) = 13.158\ ; \qquad E(33) = 30.653$$

$$E(61) = 51.874\ ; \qquad E(18) = 29.474$$

$$E(13) = 7.663\ ; \qquad E(15) = 12.968; \quad E(0) = 7.368$$

O_{ij}	E_{ij}	$\left(O_{ij} - E_{ij}\right)$	$\left(O_{ij} - E_{ij}\right)^2$	$\frac{\left(O_{ij} - E_{ij}\right)^2}{E_{ij}}$
6	13.684	−7.684	59.04386	4.31481
12	23.158	−11.158	124.501	5.376154
32	13.158	18.842	355.021	26.98138
33	30.653	2.347	5.508409	0.179702
61	51.874	9.126	83.28388	1.605503
18	29.474	−11.474	131.6527	4.466739
12	7.663	5.337	28.48357	3.717026
15	12.968	2.032	4.129024	0.318401

0	7.368	-7.368	54.28742	7.368
				54.32771

$$\chi^2 = 54.33$$

Table value of $\chi^2_{0.01}$ with $n = (r-1)(s-1) = (3-1)(3-1) = 3$ d.o.f is 11.345.

Conclusion:

Since $\chi^2 > \chi^2_{0.01}$, we reject null hypothesis. That is some relationship between fidelity and selectivity.

12. (a). (i). The following are the number of mistakes made in 5 successive days by 4 technicians working for a photographic laboratory test at a level of significance $\alpha = 0.01$. Test whether the difference among the four sample means can be attributed to chance.

Technician

I	II	III	IV
6	14	10	9
14	9	12	12
10	12	7	8
8	10	15	10
11	14	11	11

Solution:

	Technician			
	I	**II**	**III**	**IV**
	6	14	10	9
	14	9	12	12
	10	12	7	8
	8	10	15	10
	11	14	11	11
Total	49	59	55	50

Let us take the null hypothesis that there is no difference b/w four sample means.

$i.e.,\ \boldsymbol{H_0}: \ \boldsymbol{\mu_1 = \mu_2 = \mu_3 = \mu_4}$

$$Here \quad G = Grand\ total = 49 + 59 + 55 + 50 = 213$$

$$\therefore \ Correction\ factor = \frac{G^2}{N} = \frac{(213)^2}{20} = 2268.45$$

Total sum of squares $\quad SST = 2383 - 2268.45 = 114.55$

Between samples sum of squares

$$SSR = \frac{(49)^2}{5} + \frac{(59)^2}{5} + \frac{(55)^2}{5} + \frac{(50)^2}{5} - 2268.45 = 12.95$$

$$SSE = SST - SSR = 114.55 - 12.95$$

$$SSE = 101.6$$

Source of Variation	Degrees of Freedom	Sum of squares	Mean square	F-ratio
Between samples	3	12.95	4.32	$F = 0.6803$
Error	16	101.6	6.35	

The table value for $F_{(3,16)}$ at 1% level of significance is 5.29.

We accept the null hypothesis.

12. (a). (ii). The following data represents the number of units of production per day turned out by different workers using 4 different types of machines.

		Machine Type			
		A	**B**	**C**	**D**
	1	44	38	47	36
	2	46	40	52	43
Workers	**3**	34	36	44	32
	4	43	38	46	33
	5	38	42	49	39

1. Test whether the five men differ with respect to mean productivity and
2. Test whether the mean productivity is the same for the four different machine types.

Solution: Let us take the null hypothesis that

The 5 workers do not differ with respect to mean productivity

$$i.e., \quad H_{01} : \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$$

The mean productivity is the same for the four different machines.

$$i.e., \quad H_{02} : \mu_1 = \mu_2 = \mu_3 = \mu_4$$

To simplify calculation let us subtract 40 from each value, the new values are

		Machine Type				
		A	**B**	**C**	**D**	**Total**
	1	4	-2	7	-4	5
	2	6	0	12	3	21
Workers	**3**	-6	-4	4	-8	-14
	4	3	-2	6	-7	0
	5	-2	2	9	-1	8
	Total	5	-6	38	-17	20

$$Correction\ factor = C.F = \frac{G^2}{N} = \frac{(20)^2}{20} = 20$$

$$SST = Total\ sum\ of\ squares = \sum_i \sum_j y_{ij}^2 - C.F$$

$$= [(4)^2 + (-2)^2 + (7)^2 + (-4)^2 + (6)^2 + (0)^2 + (12)^2 + (3)^2 + (-6)^2 + (-4)^2 + (4)^2 + (-8)^2 + (3)^2 + (-2)^2 + (6)^2 + (-7)^2 + (-2)^2 + (2)^2 + (9)^2 + (-1)^2] - 20$$

$$= 574$$

Between workers sum of squares

$$SSC = \frac{(5)^2}{4} + \frac{(21)^2}{4} + \frac{(-14)^2}{4} + \frac{(0)^2}{4} + \frac{(8)^2}{4} - 20 = 181.5 - 20 = 161.5$$

Between machines sum of squares

$$SSR = \frac{(5)^2}{5} + \frac{(-6)^2}{5} + \frac{(38)^2}{5} + \frac{(-17)^2}{5} - 20 = 358.8 - 20 = 338.8$$

Error sum of squares

$$SSE = SST - SSC - SSR = 574 - 161.5 - 338.8 = 73.7$$

ANOVA table for two-way classification					
Source of variation	Degrees of freedom	Sum of squares (SS)	Mean sum of squares (MS)	(F-Ratio)	Table Value
B/w Row	$5-1=4$	161.5	$\frac{161.5}{4} = 40.375$	$F_1 = 6.576$	$F_{0.05}(4,12) = 3.26$
B/w Column	$4-1=3$	338.8	$\frac{338.8}{3} = 112.93$	$F_2 = 18.393$	$F_{0.05}(3,12) = 3.49$
Error	$4*3=12$	73.7	$\frac{73.7}{12} = 6.14$		

Conclusion:

1. $F_1 > F_{0.05}(4,12)$. Hence H_{01} is accepted. That is the 5 workers differ respect to mean productivity.
2. $F_2 > F_{0.05}(3,12)$. Hence H_{02} is rejected. That is the mean productivity is not the same for the four machines.

12. (b). (i). What are the basic assumptions involved in ANOVA?

Solution:

1. Each of the samples are drawn from a normal population.
2. The variances for the population from which samples have been drawn are equal.
3. The variation of each value obtained around its own grand mean should be independent for each value.

12.. (b). (ii). In a Latin square experiment given below are the yields in quintals per acre on paddy crop carried out for testing the effect of five fertilizers A, B, C, D, E. Analyze the data for variations.

B 25	A 18	E 27	D 30	C 27
A 19	D 31	C 29	E 26	B 23
C 28	B 22	D 33	A 18	E 27
E 28	C 26	A 20	B 25	D 33
D 32	E 25	B 23	C 28	A 20

Solution:

Null hypothesis: There is no significant difference between rows, columns and between the treatments.

Let us subtract 25 from each value, we get

Columns (j) / Rows (i)	1	2	3	4	5	Total
1	0	-7	2	5	2	2
2	-6	6	4	1	-2	3

3	3	-3	8	-7	2	3
4	3	1	-5	0	8	7
5	7	0	-2	3	-5	3
Total	7	-3	7	2	5	18

Treatment total $A = -7 - 6 - 7 - 5 - 5 = -30$

$B = 0 - 2 - 3 + 0 - 2 = -7$

$C = 2 + 4 + 3 + 1 + 3 = 13$

$D = 5 + 6 + 8 + 8 + 7 = 34$

$E = 2 + 1 + 2 + 3 + 0 = 8$

$$Here \quad G = 18 \quad and \quad N = 28$$

$$Correction\ factor = \frac{G^2}{N} = \frac{(18)^2}{25} = 12.96$$

$$Total\ sum\ of\ squares = \sum_i \sum_j y_{ij}^2 - C.F$$

$$SST = 496 - 12.96 = 483.04$$

Between row sum of squares

$$SSR = \frac{1}{5}(2^2 + 3^2 + 3^2 + 7^2 + 3^2) - 12.96$$

$$= 16 - 12.96 = 3.04$$

Between column sum of squares

$$SSC = \frac{1}{5}(7^2 + (-3)^2 + 7^2 + 2^2 + 5^2) - 12.96$$

$$= 27.2 - 12.96 = 14.24$$

Between treatment sum of squares

$$SSK = \frac{1}{5}[(-30)^2 + (-7)^2 + (13)^2 + (34)^2 + (8)^2] - 12.96 = 454.64$$

Error sum of squares

$$SSE = SST - SSR - SSC - SSK = 483.04 - 3.04 - 14.24 - 454.64$$

$$SSE = 11.12$$

ANOVA TABLE					Table Value
Source of variation	**D.o.f**	**Sum of squares (SS)**	**Mean Sum of squares (MS)**	**F-ratio**	
Between rows	4	3.04	0.76	$F_1 = 0.82$	$F_{0.05}(4,6) = 3.26$
Between columns	4	14.24	3.56	$F_2 == 3.83$	$F_{0.05}(4,6) = 3.26$
between treatments	4	454.64	113.66	$F_1 = 122.22$	$F_{0.05}(4,6) = 3.26$
Error	12	11.12	0.93		

Conclusion:

Since $F_1 < F_{0.05}(4,6)$ *and* $F_2 < F_{0.05}(3,6)$, we accept the null hypothesis and hence we may conclude that there is no significant difference between the rows and columns.

The calculated value of $F_3 > F_{0.05}(3,6)$, and so we conclude that the treatments are significantly different.

13. (a). (i). Find the real positive root of $3x - cosx - 1 = 0$, by Newton's method correct to 6 decimal places.
Solution:

Let $f(x) = 3x - \cos x - 1 = 0.$

Now, $f(0) = 3(0) - \cos(0) - 1 = -2 \qquad (-ve)$

$f(1) = 3(1) - \cos(1) - 1 = 1.459698 \quad (-ve)$

Therefore the root lies between 0 & 1.

Let us take $\boldsymbol{x_0 = 1}$ $\{\boldsymbol{Near\ to\ zero}\}$.

The Newton- Raphson formula is $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \ldots (1)$.

$$Let \qquad f(x) = 3x - \cos x - 1 \quad \text{and} \quad f^1(x) = 3 + \sin x$$

$$x_1 = x_0 - \left[\frac{f(x_0)}{f'(x_0)}\right] = 1 - \left[\frac{f(1)}{f'(1)}\right].$$

$$x_1 = 1 - \left[\frac{3(1) - \cos(1) - 1}{3 + \sin(1)}\right].$$

$$\boldsymbol{x_1 = 0.62002}.$$

$$x_2 = x_1 - \left[\frac{f(x_1)}{f'(x_1)}\right] = 0.62002 - \left[\frac{f(0.62002)}{f'(0.62002)}\right].$$

$$x_2 = 0.62002 - \left[\frac{3(0.62002) - \cos(0.62002) - 1}{3 + sin\ (0.62002)}\right].$$

$$\boldsymbol{x_2 = 0.60712}\ .$$

$$x_3 = x_2 - \left[\frac{f(x_2)}{f'(x_2)}\right] = 0.60712 - \left[\frac{f(0.60712)}{f'(0.60712)}\right].$$

$$x_3 = 0.60712 - \left[\frac{3(0.60712) - \cos(0.60712) - 1}{3 + sin\ (0.60712)}\right].$$

$$\boldsymbol{x_3 = 0.6071}\ .$$

The root of the equation $3x - \cos x - 1 = \boldsymbol{10}$ **is** $\boldsymbol{0.60712.}$

13. (a). (ii). Solve by Gauss Siedal method, the following system $\boldsymbol{x + y + 54z = 110}$,
$\boldsymbol{27x + 6y - z = 85}$, $\boldsymbol{6x + 15y + 2z = 72}$ correct to three decimal places.

Solution:

$$6\,x + y + 54z = 110$$
$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$

Since the diagonal elements are not dominant in the coefficient matrix, we rewrite the given equation as follows as follows

$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$
$$6\,x + y + 54z = 110$$

From the above equation, we have

$$x = \frac{1}{27}(85 - 6y + z)$$

$$y = \frac{1}{15}(72 - 6x - 2z)$$

$$z = \frac{1}{54}(110 - x - y)$$

Gauss Siedal Method:

We form the Iterations in the table

Iteration	$\boldsymbol{x}$	$\boldsymbol{y}$	$\boldsymbol{z}$
1	3.14815	3.54074	1.191317
2	2.43218	3.57204	1.92585
3	2.42569	3.57294	1.92595
4	2.42459	3.57301	1.92595
5	2.42548	3.57301	1.92595
6	2.42548	3.57301	1.92595

Hence the solution is $\boldsymbol{x} = 2.42548$, $\boldsymbol{y} = 3.57301$ $\boldsymbol{and}$ $\boldsymbol{z} = 1.92595$.

13. (b). (i). Gauss-Jordan method, find the inverse of $A = \begin{pmatrix} 4 & 1 & 2 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{pmatrix}$.

Solution:

We know that $[A, I] = [I, A^{-1}]$

Now, $[A, I] = \begin{bmatrix} 4 & 1 & 2 & \vdots & 1 & 0 & 0 \\ 2 & 3 & -1 & \vdots & 0 & 1 & 0 \\ 1 & -2 & 2 & \vdots & 0 & 0 & 1 \end{bmatrix}$

Now, we need to make $[A.I]$ as a diagonal matrix.

Fix the first row, change second and third row by using first row.

$$[A, I] \sim \begin{bmatrix} 4 & 1 & 2 & \vdots & 1 & 0 & 0 \\ 0 & 10 & -8 & \vdots & -2 & 4 & 0 \\ 0 & -9 & 6 & \vdots & -1 & 0 & 4 \end{bmatrix} \qquad \begin{matrix} R_2 \Leftrightarrow 4R_2 - 2R_1 \\ R_3 \Leftrightarrow 4R_3 - 1R_1 \end{matrix}$$

Fix the first row & second row, change third row by using second row.

$$[A, I] \sim \begin{bmatrix} 4 & 1 & 2 & \vdots & 1 & 0 & 0 \\ 0 & 10 & -8 & \vdots & -2 & 4 & 0 \\ 0 & 0 & -12 & \vdots & -28 & 36 & 40 \end{bmatrix} \qquad R_3 \Leftrightarrow 10R_3 - (-9)R_2$$

Fix the third row, change first and second row by using third row.

$$[A, I] \sim \begin{bmatrix} -48 & -12 & 0 & \vdots & 44 & -72 & -80 \\ 0 & -120 & 0 & \vdots & -200 & 240 & 320 \\ 0 & 0 & -12 & \vdots & -28 & 36 & 40 \end{bmatrix} \qquad \begin{matrix} R_1 \Leftrightarrow -12R_1 - 2R_3 \\ R_2 \Leftrightarrow -12R_2 - (-8)R_3 \end{matrix}$$

Fix the second & third row, change first by using second row.

$$[A, I] \sim \begin{bmatrix} 5760 & 0 & 0 & \vdots & -7680 & 11520 & 13440 \\ 0 & -120 & 0 & \vdots & -200 & 240 & 320 \\ 0 & 0 & -12 & \vdots & -28 & 36 & 40 \end{bmatrix} \qquad R_1 \Leftrightarrow -120R_1 - (-12)R_2$$

$$[A, I] \sim \begin{bmatrix} 1 & 0 & 0 & \vdots & \frac{-7680}{5760} & \frac{11520}{5760} & \frac{13440}{5760} \\ 0 & 1 & 0 & \vdots & \frac{-200}{-120} & \frac{240}{-120} & \frac{320}{-120} \\ 0 & 0 & 1 & \vdots & \frac{-28}{-12} & \frac{36}{-12} & \frac{40}{-12} \end{bmatrix} \qquad \begin{matrix} R_1 \Leftrightarrow R_1/960 \\ R_2 \Leftrightarrow R_2/-120 \\ R_3 \Leftrightarrow R_3/-12 \end{matrix}$$

$$A^{-1} = \begin{bmatrix} -4/3 & 2 & 7/3 \\ 5/3 & -2 & -8/3 \\ 7/3 & -3 & -10/3 \end{bmatrix}$$

Verification :

W.k.t $A\,A^{-1} = I$ $\Rightarrow \begin{bmatrix} 4 & 1 & 2 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{bmatrix} * \begin{bmatrix} -4/3 & 2 & 7/3 \\ 5/3 & -2 & -8/3 \\ 7/3 & -3 & -10/3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

13. (b). (ii). Find numerically largest Eigen value of $A = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$ and the corresponding eigenvector.

Solution: Let $X_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ be the initial vector.

Therefore,

$$A\,X_1 = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 25 \\ 1 \\ 2 \end{bmatrix} = 25 \begin{bmatrix} 1 \\ 0.04 \\ 0.08 \end{bmatrix} = 25\,X_2$$

$$A\,X_2 = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}\begin{bmatrix} 1 \\ 0.04 \\ 0.08 \end{bmatrix} = \begin{bmatrix} 25.2 \\ 1.12 \\ 1.68 \end{bmatrix} = 25.2 \begin{bmatrix} 1 \\ 0.0444 \\ 0.0667 \end{bmatrix} = 25.2\,X_3$$

$$A\,X_3 = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}\begin{bmatrix} 1 \\ 0.0444 \\ 0.0667 \end{bmatrix} = \begin{bmatrix} 25.1778 \\ 1.1332 \\ 1.7337 \end{bmatrix} = 25.1778 \begin{bmatrix} 1 \\ 0.0450 \\ 0.0688 \end{bmatrix} = 25.1778\,X_4$$

$$A\,X_4 = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}\begin{bmatrix} 1 \\ 0.0450 \\ 0.0688 \end{bmatrix} = \begin{bmatrix} 25.1826 \\ 1.135 \\ 1.7248 \end{bmatrix} = 25.1826 \begin{bmatrix} 1 \\ 0.0451 \\ 0.0685 \end{bmatrix} = 25.1826\,X_5$$

$$A\,X_5 = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}\begin{bmatrix} 1 \\ 0.0451 \\ 0.0685 \end{bmatrix} = \begin{bmatrix} 25.1821 \\ 1.1353 \\ 1.7260 \end{bmatrix} = 25.1821 \begin{bmatrix} 1 \\ 0.0451 \\ 0.0685 \end{bmatrix} = 25.1821X_6$$

$\therefore$ The dominant **Eigen value** $= 25.1821.$

Corresponding **Eigen vector** is $\begin{bmatrix} 1 \\ 0.0451 \\ 0.0685 \end{bmatrix}$.

14. (a). (i). From the following table of half-yearly maturing at premium different ages, estimate the premium for policies maturing at age 46 and 63.

Age:	45	50	55	60	65
Premium:	114.84	96.16	83.32	74.48	68.48

Solution:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
45	114.84				
		-18.68			
50	96.16		5.84		
		-12.84		-1.84	
55	83.32		4.00		0.68
		-8.84		-1.16	
60	74.48		2.84		
		-6.00			
65	68.48				

To find the value at $x = 46$:

$$y\,(x) = y\,(x_0 + ph) = y_0 + \frac{u}{1!}\,\Delta\,y_0 + \frac{u(u-1)}{2!}\,\Delta^2\,y_0 + \frac{u(u-1)(u-2)}{3!}\,\Delta^3\,y_0 + \cdots \ldots \ldots$$

$where \quad u = \frac{x-x_0}{h} \quad . \quad h = 5, \qquad Let \quad u = \left(\frac{46-45}{5}\right) = 0.2$

$$y\,(49) = 114.84 + \frac{(0.2)}{1\,!}\,(-18.68) + \frac{(0.2)[0.2-1]}{2\,!}\,(5.84) + \frac{(0.2)[0.2-1][0.2-2]}{3\,!}\,(-1.84)$$

$$+ \frac{(0.2)[0.2-1][0.2-2][0.2-3]}{4\,!}\,(0.68)$$

$$= 114.84 - 3.736 - 0.4672 - 0.08832 - 0.0228$$

$$y(46) = 110.52568$$

To find the value at $x = 63$:

$$y\,(x) = y\,(x_n + ph) \ = y_n + \frac{V}{1\,!}\,\nabla\, y_n + \frac{V(V+1)}{2\,!}\,\nabla^2 y_n + \frac{V(V+1)(V+2)}{3\,!}\,\nabla^3 y_n + \cdots \ldots \ldots$$

$$where \quad v = \frac{x - x_n}{h} = \frac{63-65}{5} = -0.4.$$

$$y\,(63) = 68.48 + \frac{(-0.4)}{1\,!}\,(-6) + \frac{(-0.4)[-0.4+1]}{2\,!}\,(2.84) + \frac{(-0.4)[-0.4+1][-0.4+2]}{3\,!}\,(-1.16)$$

$$+ \frac{(-0.4)[-0.4+1][-0.4+2][-0.4+3]}{4\,!}\,(0.68)$$

$$= 68.48 + 2.40 - 0.3408 + 0.07424 - 0.028288$$

$$y(63) = 70.58512.$$

14. (b). (i). Using Newton's divided difference formula, find the values of $f(2)$, $f(8)$ and $f(15)$ given the following table:

x:	4	5	7	10	11	13
$f(x)$:	48	100	294	900	1210	2028

Solution:

x	$f(x)$	$\Delta\, f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
4	48	$\frac{100-48}{5-4} = 52$			
			$\frac{97-52}{7-4} = 15$		
5	100	$\frac{294-100}{5-7} = 97$		$\frac{21-15}{10-4} = 1$	
			$\frac{202-97}{10-5} = 21$		0
7	294	$\frac{900-294}{10-7} = 202$		$\frac{27-21}{11-5} = 1$	
			$\frac{310-202}{11-7} = 27$		0
10	900	$\frac{1210-900}{11-10} = 310$		$\frac{33-27}{13-7} = 1$	
			$\frac{409-310}{13-10} = 33$		
11	1210	$\frac{2028-1210}{13-11} = 409$			

13	2028				

By Newton's divided difference interpolation formula

$$f(x) = f(x_0) + (x - x_0)f(x_0, x_1) + (x - x_0)(x - x_1)f(x_0, x_1, x_2) + (x - x_0)(x - x_1)(x - x_2)f(x_0, x_1, x_2, x_3)$$
$$+ (x - x_0)(x - x_1)(x - x_2)(x - x_3)f(x_0, x_1, x_2, x_3, x_4)$$

Here $x_0 = 4,\ x_1 = 5,\ x_2 = 7,\ x_3 = 10,\ x_4 = 11, x_5 = 13\ and$

$f(x_0) = 48,\ f(x_0, x_1) = 52,\ f(x_0, x_1, x_2) = 15,\ f(x_0, x_1, x_2, x_3) = 1,\ f(x_0, x_1, x_2, x_3, x_4) = 0$

$\therefore\quad f(x) = 48 + (x - 4)\,(52) + (x - 4)(x - 5)\,(15) + (x - 4)(x - 5)(x - 7)\,(1) + (x - 4)(x - 5)(x - 7)(x - 11)(0) + 0$

$= 48 + 52x - 208 + [(x^2 - 9x + 20)\,(15)] + [(x^2 - 9x + 20)(x - 7)]\,(1)$

$= 48 + 52x - 208 + 15x^2 - 135x + 300 + [x^3 - 9x^2 + 20x - 7x^2 + 63x - 140]$

$$= x^3 + x^2[15 - 9 - 7]\ + x[52 - 135 + 20 + 63] + [48 - 208 + 300 - 140]$$

$$f(x) = x^3 - x^2$$

$\therefore\quad f(2) = 2^3 - 2^2 = 4$

$f(8) = 8^3 - 8^2 = 448$

$f(15) = 15^3 - 15^2 = 3150$

14. (b). (i) Using Lagrange's interpolation formula, find $y(10)$ from the following table:

x:	5	6	9	11
y:	12	13	14	16

Solution: Given the data's are $[f(x_0) = y_0]$

x :	5 (x_0)	6 (x_1)	9 (x_2)	11 (x_3)
$f(x)$:	12 (y_0)	13 (y_1)	14 (y_2)	16 (y_3)

Lagrange's interpolation formula, we have

$$y = f(x) = \frac{(x - x_1)(x - x_2)(x - x_3)(x - x_4)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)(x_0 - x_4)}\,y_0 + \frac{(x - x_0)(x - x_2)(x - x_3)(x - x_4)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)(x_1 - x_4)}\,y_1$$

$$+ \frac{(x - x_0)(x - x_1)(x - x_3)(x - x_4)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)(x_2 - x_4)}\,y_2 + \frac{(x - x_0)(x - x_1)(x - x_2)(x - x_4)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)(x_3 - x_4)}\,y_3$$

$$y(10) = \frac{(10 - 6)(10 - 9)(10 - 11)}{(5 - 6)(5 - 9)(5 - 11)}\,(12) + \frac{(10 - 5)(10 - 9)(10 - 11)}{(6 - 5)(6 - 9)(6 - 11)}\,(13)$$
$$+ \frac{(10 - 5)(10 - 6)(10 - 15)}{(9 - 5)(9 - 6)(9 - 11)}\,(14) + \frac{(10 - 5)(10 - 6)(10 - 9)}{(11 - 5)(11 - 6)(11 - 9)}\,(16)$$

$\boldsymbol{y(10) = 14.666}$

14. (b). (ii). The table below gives the velocity V of a moving particle at time t seconds. Find the distance covered by the particle in 12 seconds and also find the acceleration at $t = 2$ seconds, using Simpson's rule.

t:	0	2	4	6	8	10	12
V:	4	6	16	34	60	94	136

Solution:

We know $\frac{ds}{dt} = v$: **and** $a = \frac{dv}{dt}$.

$$\therefore \quad S = \int v\, dt$$

To get S, we integrate v.

$S = \int_0^{12} v\, dt = \frac{2}{3}[(4+136) + 2(6+16+34+60+94)]$

$S = 522\ meters.$

Acceleration $= a = \left(\frac{dv}{dt}\right)_{t=2}$

Now we form the difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	4			
		2		
2	**6**		8	
		10		0
4	16		**8**	
		18		**0**
6	34		8	
		26		0
8	60		8	
		34		0
10	94		8	
		42		
12	136			

$$\left(\frac{dv}{dt}\right)_{t=2} = \frac{1}{h}\left[\Delta y_0 - \frac{\Delta^2 y_0}{2} + \frac{\Delta^3 y_0}{3} - \frac{\Delta^4 y_0}{4} + \cdots\right] \quad when \ \ x = x_0$$

$$\left(\frac{dv}{dt}\right)_{t=2} = \frac{1}{h}\left[10 - \frac{8}{2} + 0\right] = 3 \ \ m/s^2.$$

15. (a). (i). Using Modified Euler's method, find $y(0.2)$, $y(0.1)$ given $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$.

Solution: Given

$$y' = f(x,y) = \frac{dy}{dx} = x^2 + y^2 \qquad and \ \ y(0) = 1 \ \ \Longrightarrow \ x_0 = 0, \ \ y_0 = 1 \qquad [Since \quad y(x_0) = y_0]$$

$$\therefore \ \ h = x_1 - x_0 = 0.1 - 0 = 0.1 \qquad [Difference]$$

The Modified Euler's formula is

$$\boldsymbol{y_{n+1}(x_n + h) = y_n + h\, f\left(x_n + \frac{h}{2},\ y_n + \frac{h}{2}\, f(x_n, y_n)\right)} \quad , \ \boldsymbol{n = 0, 1, 2, \ldots..} \qquad \ldots\ldots\ldots (\mathbf{1})$$

To find $\boldsymbol{y(0.1)}$:

Put $n = 0$, equation (1) becomes

$$y_1(x_0 + h) = y_0 + h\, f\left(x_0 + \frac{h}{2},\ y_0 + \frac{h}{2}\, f(x_0, y_0)\right)$$

$We\ have \quad x_0 = 0, \ \ y_0 = 1 \ \ , \ h = 0.1 \ \ \& \ \ f(x,y) = x^2 + y^2$

$$\therefore \quad y_1(0 + 0.1) = 1 + (0.1)\ f\left(0 + \frac{0.1}{2},\ 1 + \frac{0.1}{2}\, f(0,1)\right)$$

$$y_1(0.1) = 1 + (0.1)\, f(0.05,\ \ 1 + 0.05\ [0^2 + 1^2])$$

$$= 1 + (0.1)\, f(0.05,\ \ 1 + 0.05\ [1])$$

$$= 1 + (0.1)\, f(0.05,\ \ 1.05)$$

$$= 1 + (0.1)\ [(0.05)^2 + (1.05)^2]$$

$$= 1 + (0.1)\ [1.105]$$

$$= 1 + 0.1105$$

$$y_1(0.1) = 1.1105$$

$\boldsymbol{y_1(0.1) = 1.1105}$ $\quad [\boldsymbol{y(x_1) = y_1}] \Rightarrow x_1 = \mathbf{0.1}\ \ \&\ \ \boldsymbol{y_1 = 1.1105}$

To find $\boldsymbol{y(0.2)}$:

Put $n = 1$, equation (1) becomes

$$y_2(x_1 + h) = y_1 + h\, f\left(x_1 + \frac{h}{2},\ \ y_1 + \frac{h}{2}\, f(x_1, y_1)\right)$$

$We\ have \quad x_1 = 0.1,\ \ y_1 = 1.1105\ ,\ \ h = 0.1\ \ \&\ \ f(x,y) = x^2 + y^2$

$$\therefore \quad y_2(0.1 + 0.1) = 1.1105 + (0.1)\ \ f\left(0.1 + \frac{0.1}{2},\ \ 1.1105 + \frac{0.1}{2}\, f(0.1, 1.1105)\right)$$

$$y_2(0.2) = 1.1105 + (0.1)\ \ f(0.15,\ \ 1.17266)$$

$$\boldsymbol{y_2(0.2) = 1.25026}$$

15. (a). (ii) Using Runge-Kutta method of order four solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$, given $\boldsymbol{y(0) = 1}$ at $x = 0.2$ by taking $h = 0.2$.

Solution:

$$Given \quad \frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2} = f(x,y)\ \ ,\ \ y(0) = 1 \quad \Rightarrow x_0 = 0,\ \ y_0 = 1$$

To find $\boldsymbol{h}$:

$Given\ \ y(0) = 1 \quad \Rightarrow \quad y(x_0) = y_0 \quad \Rightarrow x_0 = 0,\ \ y_0 = 1$

We need to find $y(0.2) \Rightarrow y(x_1) = y_1 \quad \Rightarrow x_1 = 0.2$

$\therefore\ \ h = x_1 - x_0 = 0.2 - 0.0 = 0.2$

$\therefore\ \ \boldsymbol{h = 0.2}$

To find $\boldsymbol{y(0.2)}$: $\quad \left[\boldsymbol{x_0 = 0.0,\ \ y_0 = 1,\ \ h = 0.2,\ \ f(x,y) = \frac{y^2 - x^2}{y^2 + x^2}}\right]$

$$k_1 = h\, f(x_0, y_0) = 0.2\, f(0,\ \ 1\ ) = 0.2\left[\frac{(1)^2 - (0)^2}{(1)^2 + (0)^2}\right] = 0.2[1]$$

$\boldsymbol{k_1 = 0.2}$

$$k_2 = h\, f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = 0.2\, f\left(0 + \frac{0.2}{2},\ \ 1 + \frac{0.2}{2}\ \right) = 0.2\, f(0.1,\ \ 1.1)$$

$$k_2 = 0.2\ \left[\frac{(1.1)^2 - (0.1)^2}{(1.1)^2 + (0.1)^2}\right]$$

$\boldsymbol{k_2 = 0.1967213}$

$$k_3 = h\,f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right) = 0.2\,f\left(0 + \frac{0.2}{2}, 1 + \frac{0.1967213}{2}\right) = 0.2f(0.1,\ \ 1.0983606)$$

$$k_3 = 0.2\ \left[\frac{(1.0983606)^2 - (0.1)^2}{(1.0983606)^2 + (0.1)^2}\right]$$

$$\boldsymbol{k_3 = 0.1967}$$

$$k_4 = h\,f(x_0 + h, y_0 + k_3) = 0.2\ \ f(0 + 0.2,\ \ 1 + 0.1967) = 0.2\ \left[\frac{(0.1967)^2 - (0.2)^2}{(0.1967)^2 + (0.2)^2}\right]$$

$$\boldsymbol{k_4 = 0.1891}$$

$$\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = \frac{1}{6}(0.2 + 2(0.1967213) + 2(0.1967) + 0.1891)$$

$$\boldsymbol{\Delta y = 0.19598}$$

$$y(x+h) = y(x) + \Delta y \quad \Longrightarrow \ y(0.0 + 0.2) = y(0) + 0.44321 = 1 + 0.19598$$

$$\boldsymbol{y(0.2) = 0.19598}$$

15. (b). (i). Using Taylor method, compute $y(0.2)$ and $y(0.4)$ correct to 4 decimal places given $\frac{dy}{dx} = 1 - 2\,x\,y$ and $y(0\,) = 0$, by taking $h = 0.2$.

Solution:

$$Given \quad \frac{dy}{dx} = y' = 1 - 2\,x\,y \quad \& \quad y(0) = 0 \ \Longrightarrow \ x_0 = 0, \qquad y_0 = 0 \qquad Since\ \ [y(x_0) = y_0]$$

Taylor series formula is

$$y(x) = y_0 + \frac{(x - x_0)}{1\,!}\,y_0' + \frac{(x - x_0)^2}{2\,!}\,y_0'' + \frac{(x - x_0)^3}{3\,!}\,y_0''' + \frac{(x - x_0)^4}{4\,!}\,y_0^{iv} + \frac{(x - x_0)^5}{5\,!}\,y_0^{v} + \cdots \quad \ldots..(1)$$

$y' = 1 - 2\,x\,y$	$y_0' = 1 - 2\,x_0\,y_0 = 1 - 2(0)(0)$	$y_0' = 1$
$y'' = 0 - 2\,x\,y' - 2y$	$y_0'' = -2\,x_0\,y_0' - 2y_0$ $= -2(0)(1) - 2(0)$	$y_0'' = 0$
$y''' = -2\,x\,y'' - 2y' - 2y'$ $y''' = -2\,x\,y'' - 4\,y'$	$y_0''' = -2\,x_0\,y_0'' - 4\,y_0'$ $= -2(0)(0) - 4(1)$	$y_0''' = -4$
$y'^{v} = -2\,x\,y''' - 2\,y'' - 4\,y''$ $y'^{v} = -2\,x\,y''' - 6\,y''$	$y_0'^{v} = -2\,x_0\,y_0''' - 6\,y_0''$ $= -2(0)(-4) - 6(0)$	$y_0'^{v} = 0$
$y^{v} = -2\,x\,y'^{v} - 2\,y''' - 6\,y'''$ $y'^{v} = -2\,x\,y'^{v} - 8\,y'''$	$y_0'^{v} = -2\,x_0\,y_0'^{v} - 8\,y_0'''$ $= -2(0)(0) - 8(-4)$	$y_0'^{v} = 32$

Therefore equation (1) becomes,

$$y(x) = y_0 + \frac{(x - x_0)}{1\,!}\,y_0' + \frac{(x - x_0)^2}{2\,!}\,y_0'' + \frac{(x - x_0)^3}{3\,!}\,y_0''' + \frac{(x - x_0)^4}{4\,!}\,y_0^{iv} + \frac{(x - x_0)^5}{5\,!}\,y_0^{v} + \cdots \qquad \ldots..(1)$$

$$= 0 + \frac{(x - 0)}{1}\,(1) + \frac{(x - 0)^2}{2}\,(0) + \frac{(x - 0)^3}{6}\,(-4) + \frac{(x - 0)^4}{24}\,(0) + \frac{(x - 0)^5}{120}\,(32) \ldots$$

$$y(x) = x\ + \frac{x^3}{6}\,(-4) + \frac{x^5}{120}\,(32)$$

To find $y(0.2)$:

$$\therefore \quad y(0.2) = 0.2 + \frac{(0.2)^3}{6}(-4) + \frac{(0.2)^5}{120}(32) = 0.2 - 0.005333 + 0.00008533$$

$$\therefore \quad \boldsymbol{y(0.2) = 0.194752}$$

To find $y(0.4)$:

$$\therefore \quad y(0.4) = 0.4 + \frac{(0.4)^3}{6}(-4) + \frac{(0.4)^5}{120}(32) \quad = 0.4 - 0.0426667 + 0.002730667$$

$$\therefore \quad \boldsymbol{y(0.4) = 0.360063.}$$

15. (b). (ii). Using Adam's method find $y(0.4)$ given $\frac{dy}{dx} = \frac{1}{2}xy$, $y(0) = 1$, $y(0.1) = 1.01$, $y(0.2) = 1.022$, $y(0.3) = 1.023$.

Solution: Given

$$y' = f(x,y) = \frac{dy}{dx} = \frac{1}{2}xy$$

$y(0) = 1$	$x_0 = 0$	$y_0 = 1$
$y(0.1) = 1.01$	$x_1 = 0.1$	$y_1 = 1.01$
$y(0.2) = 1.022$	$x_2 = 0.2$	$y_2 = 1.022$
$y(0.3) = 1.023$	$x_3 = 0.3$	$y_3 = 1.023$

$\boldsymbol{Here \quad h = 0.1}$ **and** $\boldsymbol{n = 3 \quad [Highest\ value\ of\ x\ is\ x_3. \ \therefore n = 3]}$

The Adam's Predictor formula is

$$y_{n+1,P}(x_n + h) = y_n + \frac{h}{24}[55y'_n - 59y'_{n-1} + 37\, y'_{n-2} - 9\, y'_{n-3}] \quad \dots\dots (1)$$

Put n=3 in equation (1), we have

$$y_{4,P}(x_3 + h) = y_3 + \frac{h}{24}[55y'_3 - 59y'_2 + 37\, y'_1 - 9\, y'_0] \qquad \dots\dots (2)$$

$Given \quad y' = x^2(1 + y)$

$x_0 = 0$	$y_0 = 1$	$y'_0 = \left(\frac{1}{2}\right)(x_0 y_0)$	$y'_1 = 0$
$x_1 = 0.1$	$y_1 = 1.01$	$y'_1 = \left(\frac{1}{2}\right)(0.1 * 1.01)$	$y'_1 = 0.0505$
$x_2 = 0.2$	$y_2 = 1.022$	$y'_2 = \left(\frac{1}{2}\right)(0.2 * 1.022)$	$y'_2 = 0.1022$
$x_3 = 0.3$	$y_3 = 1.023$	$y'_3 = \left(\frac{1}{2}\right)(0.3 * 1.023)$	$y'_3 = 0.15345$

Equation (2) becomes

$$y_{4,P}(1.3 + 0.1) = 1.023 + \frac{0.1}{24}[55(0.15345) - 59(0.1022) + 37\,(0.0505) - 9\,(0)]$$

$$\boldsymbol{y_{4,P}(0.4) \;\; = 1.0408 \quad [y(x_4) = y_4, \quad x_4 = 1.4 \quad \& \quad y_4 = 2.5721\,]}$$

The Adams's Corrector formula is

$$y_{n+1,C}(x_n + h) = y_n + \frac{h}{24}[\,9\,y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2}] \quad \dots\dots (3)$$

Put n=3 in equation (3), we have

$$y_{4,C}(x_3 + h) = y_3 + \frac{h}{24}[\,9\,\boldsymbol{y'_4} + 19y'_3 - 5y'_2 + y'_1] \quad \dots\dots (4)$$

$x_4 = 0.4$	$y_4 = 1.0408$	$y'_4 = \left(\frac{1}{2}\right)(0.4 * 1.0408)$	$y'_4 = 0.20816$

Equation (4) becomes

$$y_{4,C}(1.3 + 0.1) = 1.023 + \frac{0.1}{24}[\,9\,(0.20816) + 19\,(0.15345) - 5\,(0.1022) + (0.0505)]$$

$$\boldsymbol{y_{4,C}(0.4) = 1.0410}$$

Result:

$$\boldsymbol{y_{4,P}(0.4) = 1.0408} \quad \& \quad \boldsymbol{y_{4,C}(0.4) = 1.0410}$$