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AU- chennai - Nov/Dec 2011

Numerical Methods

Part-A

1) Solve $e^x - 3x = 0$ by the method of iteration.

Soln: $f(x) = e^x - 3x$

$f(0) = 1$

$f(1) = -0.2871$

∴ The root lies between 0 and 1

Consider $e^x - 3x = 0 \Rightarrow x = \frac{e^x}{3}$

$\therefore x_{n+1} = \frac{e^{x_n}}{3}$

Let $x_0 = 1$

$x_1 = 0.90609$

$x_2 = 0.82487$

$x_3 = 0.76052$

$x_4 = 0.71312$

$x_5 = 0.68011$

$x_6 = 0.65803$

$x_7 = 0.64366$

$x_8 = 0.63447$

$x_9 = 0.62867$

$x_{10} = 0.62503$

$x_{11} = 0.62276$

$x_{12} = 0.62135$

$x_{13} = 0.62047$

$x_{14} = 0.61993$

$x_{15} = 0.61959$

∴ The root is $x = 0.619$.

2) Using Newton's method, find the root between 0 and 1 of $x^3 = 6x - 4$.

Soln: $f(x) = x^3 - 6x + 4 = 0$

$f(0) = 4$

$f(1) = -1$

∴ The root lies between 0 and 1

Let $x_0 = 0.5$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{(x_n^3 - 6x_n + 4)}{3x_n^2 - 6}$$

$$x_{n+1} = \frac{2x_n^3 - 4}{3x_n^2 - 6}$$

$x_0 = 0.5$

$x_1 = 0.667$

$x_2 = 0.667$

$\therefore$ The root is $x = 0.667$.

3) State Lagrange's formula for unequal intervals.

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)\cdots(x_0-x_n)} y_0$$

$$+ \cdots + \frac{(x-x_0)(x-x_1)(x-x_2)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)(x_n-x_2)\cdots(x_n-x_{n-1})} y_n .$$

4) Define cubic spline.

$$S_i(x) = \frac{1}{6h}\left[(x_i - x)^3 M_{i-1} + (x - x_{i-1})^3 M_i\right]$$

$$+ \frac{1}{h}\left[(x_i - x)\left(y_{i-1} - \frac{h^2}{6} M_{i-1}\right)\right]$$

$$+ \frac{1}{h}\left[(x - x_{i-1})\left(y_i - \frac{h^2}{6} M_i\right)\right]$$

5) State Simpson's 1/3 rule.

$$\int_{x_0}^{x_n} f(x)\,dx = \frac{h}{3}\left[(y_0 + y_n) + 2(y_2 + y_4 + y_6 + \cdots) + 4(y_1 + y_3 + y_5 + \cdots)\right]$$

b) Write down two point Gaussian quadrature formula.

$$\int_{-1}^{1} f(t)\,dt = f(1/\sqrt{3}) + f(-1/\sqrt{3})$$

7) State Euler's method to solve $\frac{dy}{dx} = f(x,y)$ with $y(x_0) = y_0$

Soln: $y_{n+1} = y_n + h f(x_n, y_n)$

8) State Adam's predictor - corrector formulae.

Soln:

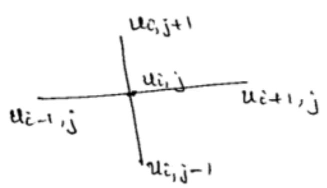
Predictor formula: $y_{n+1,p} = y_n + \frac{h}{24}(55y_n' - 59y_{n-1}' + 37y_{n-2}' - 9y_{n-3}')$

Corrector formula: $y_{n+1,c} = y_n + \frac{h}{24}(9y_{n+1}' + 19y_n' - 5y_{n-1}' + y_{n-2}')$

9) Classify the PDE $y(x_0) = y_0$.

10) State standard five point formula with relevent diagram.

$$u_{i,j} = \frac{1}{4}\left[u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1}\right]$$



Part - B

11) a) (i) Find an iterative formula to find the reciprocal of a given number N and hence find the value of $\frac{1}{19}$.

Soln. Let $x = \frac{1}{N}$

$N = 1/x$.

$f(x) = \frac{1}{x} - N = 0, \quad f'(x) = \frac{-1}{x^2}$

By iterative formula,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{\left(\frac{1}{x_n} - N\right)}{-1/x_n^2}$$

$x_{n+1} = 2x_n - N x_n^2$ is the iterative formula.

To find the value of $\frac{1}{19}$:

let $N = 19$

and $\frac{1}{20} = 0.05$

let $x_0 = 0.05$

$x_1 = 0.0525$

$x_2 = 0.05263125$

$x_3 = 0.0526315789$

$x_4 = 0.0526315789$

$\therefore \frac{1}{19} = 0.0526315789$.

(ii) Apply Gauss - Jordan method to find the solution of the following system $10x + y + z = 12$, $2x + 10y + z = 13$, $x + y + 5z = 7$.

Soln:

The argumented matrix is $(A, B) = \begin{pmatrix} 10 & 1 & 1 & 12 \\ 2 & 10 & 1 & 13 \\ 1 & 1 & 5 & 7 \end{pmatrix}$

$$\sim \begin{pmatrix} 10 & 1 & 1 & 12 \\ 0 & 49 & 4 & 53 \\ 0 & 9 & 49 & 58 \end{pmatrix}$$

$$\sim \begin{pmatrix} 10 & 0 & 45 & 535 \\ 0 & 49 & 4 & 53 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

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$$\sim \begin{pmatrix} 490 & 0 & 0 & 490 \\ 0 & 49 & 0 & 49 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

$490x = 490 \Rightarrow x = 1$

$49y = 49 \Rightarrow y = 1$

$z = 1 \Rightarrow z = 1$

The soln is $x=1, y=1, z=1$.

b) (i) Solve, by Gauss-seidal method, the following system

$28x+4y-z=32,\quad x+3y+10z=24,\quad 2x+17y+4z=35$

Soln:

Refer AUQ : May | June 2007, Q.No 11)(a)(ii)

(ii) Find the largest eigen value of $\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ by using power method.

Refer AUQ : May | June 2007, Q.No 11) (b) (ii)

12)a) (i) The population of a town is as follows

x: year	:	1941	1951	1961	1971	1981	1991
Y: Population in thousands	:	20	24	29	36	46	51

Estimate the population increase during the period 1946 to 1976.

Soln:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1941	20					
		4				
1951	24		1			
		5		1		
1961	29		2		0	
		7		1		−9
1971	36		3		−9	
		10		−8		
1981	46		−5			
		5				
1991	51					

$$n = \frac{x - x_0}{h} = \frac{1946-1941}{10} = 0.5$$

By Newton's forward formula,

$$y(x) = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{2!}\Delta^3 y_0 + \cdots$$

$$= 20 + (0.5)(4) + \frac{(0.5)(.5-1)}{2}(1) + \frac{(0.5)(0.5-1)(0.5-2)}{6} \times 1$$

$$+ \frac{(0.5)(0.5-1)(0.5-2)(0.5-3)}{24}(0) + \frac{(0.5)(0.5-1)(0.5-2)(0.5-3)(0}{120}$$

$$= 20 + 2 - 0.125 + 0.0625 - 0.24609$$

$$\boxed{y(x) = 21.69}$$

By Newton's Backward formula, $n = \frac{x - x_n}{h} = \frac{1976-1991}{10} =$

$$y(x) = y_n + n\nabla y_n + \frac{n(n+1)}{2!}\nabla^2 y_n + \frac{n(n+1)(n+2)}{3!}\nabla^3 y_n + \cdots$$

$$= 51 + (-1.5)(5) + \frac{(-1.5)(-1.5+1)}{2}(-5) + \frac{(-1.5)(-1.5+1)(-1.5+2)}{6}(-8$$

$$+ \frac{(-1.5)(-1.5+1)(-1.5+2)(-1.5+3)}{24}(-9)$$

$$+ \frac{(-1.5)(-1.5+1)(-1.5+2)(-1.5+3)(-1.5+4)}{120} \times (-9)$$

$$\boxed{y(1976) = 40.808596}$$

$\therefore$ Increase in population during the period 1946 to 1976 is

$$= 40.808596 - 21.69$$

$$= 19.4 \text{ lakh}$$

b) Determine $f(x)$ as a polynomial in x for the following data, using Newton divided difference formula. Also find $f(2)$

x :	-4	-1	0	2	5
$f(x)$:	1245	33	5	9	1335

(6)

Soln:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
-4	1245				
		-404			
-1	33		94		
		-28		-14	
0	5		10		3
		2		13	
2	9		88		
		442			
5	1335				

By divided difference formula,

$$y(x) = y_0 + (x-x_0)\Delta y_0 + (x-x_0)(x-x_1)\Delta^2 y_0 + (x-x_0)(x-x_1)(x-x_2)\Delta^3 y_0 + \dots$$

$$= 1245 + (x+4)(-404) + (x+4)(x+1)\,94 + (x+4)(x+1)x(-14)$$

$$+ (x+4)(x+1)\,x\,(x-2)(3)$$

$$\boxed{y(2) = 9}$$

1)a)

(i) Find the first two derivatives of $x^{1/3}$ at $x=50$ and $x=56$, for the given table

x :	50	51	52	53	54	55	56
$y = x^{1/3}$:	3.684	3.7084	3.7325	3.7563	3.7798	3.803	3.8259

Soln:

By Newton's forward formula

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{\Delta^2 y_0}{2} + \frac{\Delta^3 y_0}{3} - \frac{\Delta^4 y_0}{4} + \dots\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \dots\right]$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
50	3.684			
		0.0244		
51	3.7084		-0.0003	
		0.0241		0
52	3.7325		-0.0003	
		0.0238		0
53	3.7563		-0.0003	
		0.0235		0
54	3.7798		-0.0003	
		0.0232		0
55	3.803		-0.0003	
		0.0229		
56	3.8255			

Forward formula,

$$\left.\frac{dy}{dx}\right|_{x=50} = \frac{1}{1}\left[0.0244 - \frac{1}{2}(-0.0003)\right] + \cdots = 0.02455$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=50} = \frac{1}{1^2}\left[-0.0003 - 0\right] = -0.0003$$

Backward formula

$$\left.\frac{dy}{dx}\right|_{x=x_n} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \cdots\right]$$

$$\left.\frac{dy}{dx}\right|_{x=56} = \frac{1}{1}\left[0.0229 + \frac{1}{2}(-0.0003)\right] = 0.02275$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=x_n} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \cdots\right]$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=56} = \frac{1}{1}\left[-0.0003\right] = -0.0003$$

(or).

b) Evaluate $I = \int_0^6 \frac{dx}{1+x}$ by using (i) direct method (ii) Trapezoidal rule (iii) Simpson's one-third rule (iv) Simpson's three-eight rule

Soln:

By integration

$$I = \int_0^6 \frac{dx}{1+x} = \log(1+x)\Big]_0^6 = \log 7 - \log 1 = 1.946$$