1. Show that $(\mathrm{P} \to (\mathrm{Q} \to \mathrm{R})) \to ((\mathrm{P} \to \mathrm{Q}) \to (\mathrm{P} \to \mathrm{R}))$ is a tautology.
2. Without using truth table show that $\mathrm{P} \to (\mathrm{Q} \to \mathrm{P}) \Rightarrow \sim \mathrm{P} \to (\mathrm{P} \to \mathrm{Q})$
3. Prove that $p, p \to q, q \to r \Rightarrow r$
4. When do you say that two compound propositions are equivalent?
5. Using truth table, show that the proposition $P \vee \sim (P \wedge Q)$ is a tautology.
6. If seven colours are used to paint 50 bicycles, then show that at least 8 bicycles will be the same colour.
7. Define the rule of universal specification.
8. Give an indirect proof of the theorem "If $3n + 2$ is odd, then n is odd".
9. State Pigeonhole principle.
10. Find the recurrence relation for the Fibonacci sequence.
11. Using indirect method of proof, derive $p \to \sim s$ from the premises $p \to (q \vee r), q \to \sim p, s \to \sim r$ and p.
12. Show that the hypothesis, "It is not sunny this afternoon and it is colder than yesterday", " we will go swimming only if it is sunny", "If we do not go swimming, then we will take a canoe trip", " If we take a canoe trip, then we will be home by sunset" lead to the conclusion " we will be home by sunset".
13. Show that $\mathrm{P} \vee (\mathrm{Q} \wedge \mathrm{R})$ and $(\mathrm{P} \vee \mathrm{Q}) \wedge (\mathrm{P} \vee \mathrm{R})$ are logically equivalent.
14. Obtain the principal disjunctive normal form and principal conjunction form of the statement
$$\mathrm{p} \vee (\sim \mathrm{p} \to (\mathrm{q} \vee (\sim \mathrm{q} \to \mathrm{r})))$$
15. Without using truth table find the PCNF and PDNF of
$$\mathrm{P} \to (\mathrm{Q} \wedge \mathrm{P}) \wedge (\sim \mathrm{P} \to (\sim \mathrm{Q} \wedge \sim \mathrm{R}))$$
16. Prove that the premises $a \to (b \to c), d \to (b \wedge \sim c)$ and $(a \wedge d)$ are inconsistent**.**
17. Use the indirect method to prove that the conclusion $\exists z(Q(z))$ follows from the premises $\forall x(P(x) \to Q(x))$ and $\exists y(P(y))$.
18. Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction.
19. Use mathematical induction to prove the inequality $n < 2^n$ for all positive integer n.
20. Use Mathematical induction show that
$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$
21. Find the number of integers between 1 and 250 both inclusive that are divisible by any of the integers 2,3,5 and 7.

22. Find the number of distinct permutations that can be formed from all the letters of each word
(1) RADAR (2) UNUSUAL (3) MATHEMATICS (4) STATISTICS

23. A box contains six white balls and five red balls. Find the number of ways four balls can be drawn from the box if
(1) They can be any colour
(2) Two must be white and two red

(3) They must all be the same colour.

24. There are 2500 students in a college, of these 1700 have taken a course in C, 1000 have taken a course in Pascal and 550 have taken a course in Networking. Further 750 have taken courses in both C and Pascal. 400 have taken courses in both C and Networking, and 275 have taken courses in both Pascal and Networking. If 200 of these students have taken courses in C, Pascal and Networking then

(1) How many of these 2500 students have taken a course in any of these three courses C, Pascal and Networking?

(2) How many of these 2500 students have not taken a course in any of these three courses C, Pascal and Networking?

25. Using method of generating function to solve the recurrence relation

$a_n + 3a_{n-1} - 4a_{n-2} = 0; n \geq 2$, given that $a_0 = 3$ and $a_1 = -2$.

26. Using the generating function, solve the difference equation

$$\mathrm{y}_{\mathrm{n}+2} - \mathrm{y}_{\mathrm{n}+1} - 6\mathrm{y}_{\mathrm{n}} = 0, \mathrm{y}_1 = 1, \mathrm{y}_0 = 2$$