

ENGINEERING MECHANICS

TWO- MARKS

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MODULE I - BASICS AND STATICS OF PARTICLES

1. UNITS

<i>QUANTITY</i>	<i>mass</i>	<i>time</i>	<i>current</i>	<i>temp</i>	<i>intensity</i>	<i>length</i>
<i>UNIT</i>	<i>kilogram</i>	<i>second</i>	<i>ampere</i>	<i>Kelvin</i>	<i>candela</i>	<i>Meter</i>

2. PARELLOGRAM LAW

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

Where R_ Resultant

P_ 1stforce

Q_ 2ndforce

θ _ Angle between two forces

3. SCALAR QUANTITY

“It’s one having only magnitude”

4. VECTOR QUANTITY

“It’s one having both magnitude and direction”

5. POSITION VECTOR

$$R = \sqrt{x^2 + y^2 + z^2}$$

6. DOT PRODUCT

.	I	J	k
I	1	0	0
j	0	1	0
k	0	0	1

7. CROSS PRODUCT

X	I	J	k
I	0	K	-j
j	-k	0	I
k	J	-I	0

8, LAMIS THEOREM

If three coplanar forces are acting at a point in equilibrium

Then each force is directly proportional to sine of opposite two forces.

$$P/\sin \alpha = Q/\sin \beta = R/\sin \phi$$

9. LAW OF MECHANICS1. A particle remains in its position if resultant force is zero

2. If the resultant force is not zero then acceleration is proportional to resultant force $F = ma$.

3. Action and reaction forces are in same line of action, equal in equal in magnitude but opposite in direction.

10. NEWTONS LAW OF GRAVITATION

The law states that two particles of mass `m` and mutually attracted
By two opposite forces F,-F then magnitude of F is given by
 $F = G Mm/ R^2$ Newton

11. PRINCIPLE OF TRANSMISSIBILITY

Principle of transmissibility states that “ the motion of a rigid body
remains unchanged if a force acting on a point is replaced by
another force having same magnitude and direction in the same
line of action.

12. RESULTANT FOR MORE THAN 2 FORCES

For a system when resolving

$$F_h = F \cos A, F_v = F \sin B$$

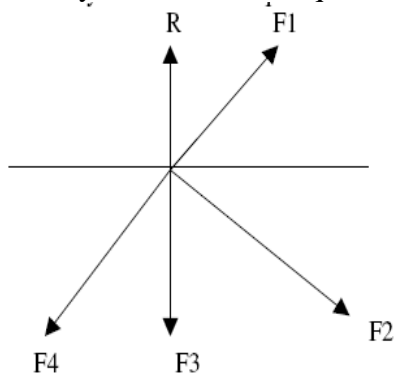
The resultant force is $R = \sqrt{(\sum h)^2 + (\sum v)^2}$

13. EQUILIBRANT

A force E with same magnitude and same line of action but opposite
Direction is called equilibrant.

14. EQUILIBRIUM

The equilibrant used to arrest the movement of the particle, then the
The body is said to be in equilibrium. Here Resultant = 0.



15. CONDITIONS FOR EQUILIBRIUM

For collinear forces, $\sum H = 0, \sum V = 0$

For concurrent forces, $\sum H = 0, \sum V = 0$

16. FREE BODY

A body which has been isolated from surroundings is called
Free body.

17. FREE BODY DIAGRAM

The sketch showing all the forces and moments acting on the
Free body is called free body diagram

18. HORIZONTAL & VERTICAL COMPONENTS IN EACH QUADRANT

F_h F_v

1st +VE +VE

2nd -VE +VE

3rd -VE -VE

4th +VE -VE

19. FORCES IN THREE DIMENSION

Consider a system of force in three dimensions

So that

$$F = F \cos \theta_x + F \sin \theta_y + F \sin \theta_z$$

$$\theta_x = \cos^{-1} (F_x / F)$$

$$\theta_y = \cos^{-1} (F_y / F)$$

$$\theta_z = \cos^{-1} (F_z / F)$$

20. CONDITIONS

$$\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1$$

$$l^2 + m^2 + n^2 = 1$$

21. RESULTANT

$$R = R_x i + R_y j + R_z k$$

$$R = \sqrt{(R_x^2 + R_y^2 + R_z^2)}$$

Where

$$\theta_x = \cos^{-1} (R_x / R), \theta_y = \cos^{-1} (R_y / R)$$

$$\theta_z = \cos^{-1} (R_z / R)$$

22. POSITION VECTOR If Coordinates of A is (X₁ , Y₂ , Z₃)

Coordinates of B is (X₂ , Y₂ , Z₃)

Then $r_{ab} = xi + yj + zk$

Where

$$X = x_2 - x_1$$

$$Y = y_2 - y_1$$

$$Z = Z_2 - z_1$$

23. FORCE VECTOR

$$F_{ab} = F_{ab} \cdot (\text{UNIT VECTOR})$$

where

$$\text{Unit vector} = \frac{F_x i + F_y j + F_z k}{\sqrt{F_x^2 + F_y^2 + F_z^2}}$$

24. EQUATION OF FORCE IN THREE DIMENSION

$$_F x = 0 \quad _F y = 0 \quad _F z$$

MODULE II - STATICS OF RIGID BODIES

1. Rigid bodies When a body is not subjected to collinear or concurrent force system, then the body is to be idealized as a rigid body.

2. Moment of force Moment of force is defined as the product of the force and the perpendicular distance of the line of action of force from the point. It's unit is N-m. $M = F \cdot r$ N-m

3. Clockwise moment _ positive sign
Anticlockwise _ negative sign

4. Varignon's theorem of moment :
The algebraic sum of moments of any number of forces about any point in their plane is equal to moment of their resultant about the same point.

(ie.) $F_1d_1 + F_2d_2 + F_3d_3 + \dots = R \times d$

5. Moment passing through the reference point is zero.

6. Resultant of non-parallel , non-concurrent and coplanar forces is given by,

$$R = \sqrt{(\sum H)^2 + (\sum V)^2}$$

where $\sum H = F_{h1} + F_{h2} + F_{h3} + \dots$
 $\sum V = F_{v1} + F_{v2} + F_{v3} + \dots$
 $\theta = \tan^{-1} (\sum H / \sum V)$

7. In the parallel force system, resultant should be in between the two forces and parallel to the forces.

8. Force – couple system

Couple: Two forces F and $-F$ having the same magnitude, parallel line of action but in opposite direction are said to form a couple.

9. For magnitude of the resultant force, magnitude of couple is not considered.

10. In a couple, sum of the moments of two forces about any point is same in magnitude and direction.

11. In general, moment of couple = $F \times$ arm length

12. Types of couple

- a. Clockwise couple
- b. Anticlockwise couple

13. Equilibrium of rigid body in two dimension

For the equilibrium state of rigid body, the resultant force and the resultant force and the resultant moment with respect to any point is zero.

$$\sum H = 0 ; \sum V = 0 ; \sum M = 0$$

14. For parallel forces acting on a rigid body

$$\sum V = 0 ; \sum M = 0$$

15. For inclined forces acting on a rigid body

$$\sum V = 0 ; \sum H = 0 ; \sum M = 0$$

16. Difference between moment and couple

The couple is a pure turning effect, which may be moved anywhere in its own plane whereas, moment of force must include a description of the reference axis about which the moment is taken.

17. Action

The self weight of a body acting vertically downwards is known as action.

18. Reaction

The resulting force against the action acting vertically upwards is known as reaction. It is developed at the support.

19. Support reaction

The resistance force on the beam against the applied load at support is called support reaction.

20. Types of support

- i. Roller support
- ii. Hinged support
- iii. Fixed support

21. Roller support has only one reaction

22. Hinged support is also called pin-joint support.

23. Roller and hinged supports can resist only displacement but rotation of beam is not resisted by both the supports.

24. Roller support has the known line of action i.e. always normal to the plane of rollers.

25. Hinged support has an unknown line of action i.e. at any angle,

with the horizontal

26. Vertical reactions are assumed upwards and horizontal reactions outwards. While solving, if negative value is obtained, then the direction is reversed.

27. Reaction

$$R_A = \sqrt{H_A^2 + V_A^2}$$
$$\theta = \tan^{-1}(V_A/H_A)$$

28. Fixed support

It has three reactions at a point.

29. Types of load on a beam

1. Point load
2. Uniformly distributed load (UDL)
3. Uniformly varying load (UVL)

30. Uniformly distributed load (UDL)

This load is applied all over the area uniformly. It acts exactly at the mid-point of the body.

Uniformly varying load (UVL)

The total load acts at the centroid of the triangle.

31. If the beam is supported at the two ends, it is known as simply supported beam (SS beam).

32. Supported reactions of trusses

Trusses are made up of several bars and joints. It is subjected to point loads at the joints.

33. Neglect the self weight of the truss, if it is given in the problem.

34. If truss is given in a problem, modify the truss configuration to beam configuration and then solve the problem.

35. To find the moment of force about an axis, the moment of the force about any point, lying on the particular axis should be known.

36. Moment of force about a point is a vector quantity, where as the moment of force about the axis passing through that point is a scalar quantity.

37. If all the forces are in the XZ plane, parallel to Y-axis, then scalar approach can be used.

MODULE-III- PROPERTIES OF SURFACES AND SOLIDS

1. CENTROID

CENTROID is the point where the entire weight of the body

will act in Two Dimensional. In this case mass of the body is neglected.

2. CENTER OF GRAVITY

CENTER OF GRAVITY is the point where entire weight of the body will act in Three Dimensional. Here weight of the body is considered.

3. MOMENT OF INERTIA

- i. MOMENT OF INERTIA about any axis in the plane may be defined as the sum of the product of each elemental area and the square of the perpendicular distance between the element and the axis.
- ii. M.I. is also called as SECOND MOMENT OF AN AREA.
- iii. M.I. of an area about x-axis is represented as I_{xx} and about y -axis is represented by I_{yy} .
Unit of moment of inertia is mm^4 .

4. PERPENDICULAR AXIS THEOREM

The perpendicular axis theorem states that the moment of Inertia of a plane area about an axis perpendicular to the plane and Passing through the intersection of the other two axes xx and yy Contained by the plane is equal to the moment if inertia about xx and yy.

$$I_{zz} = I_{xx} + I_{yy}$$

5. POLAR MOMENT OF INERTIA:(I_p)

The polar moment of inertia is the moment of inertia about the Pole axis (I_{zz}) from the distance R.

$$I_{zz} = I_{xx} + I_{yy}$$

This is used for determination of study of shaft under twisting.

6.RADIUS OF GYRATION

If the plane area can be represented as plane figure parallel to xx axis and parallel to yy axis . r_x is the distance from yy and r_y is the distance from xx then

$$\text{MI along x axis} = I_{xx} = \int y^2 dA$$

$$\text{Radius of Gyration } r_{xx} = \sqrt{(I_{xx}/A)}$$

$$\text{MI along y axis} = I_{yy} = \int x^2 dA$$

$$\text{Radius of Gyration } r_{yy} = \sqrt{(I_{yy}/A)}$$

This will be used for determine the stability of columns.

7.PARALLEL AXIS THEOREM

The theorem states that the moment of inertia of the plane area about any axis parallel the centroidal axis (I_{AB}) is equal to moment of inertia of the area about the centroidal axis (I_{CG}) added to product of area (A) and square of the perpendicular distance between the two axes (h). $I_{ab} = I_{CG} + Ah^2$
 I_{CG} is also called as self inertia.

8. PRODUCT OF MOMENT OF INERTIA

The product of inertia for the elementary area is defined as dI_{xy} and is equal to $(xy) (dA)$. Thus, for the entire area A , the product of inertia is, $I_{xy} = \int xy \, dA$

$$I_{xy} = A_1 X_1' Y_1' + A_2 X_2' Y_2' + \dots + A_n X_n' Y_n'$$

Where $X_1' = \bar{X} - X_1$, $X_2' = \bar{X} - X_2$

$Y_1' = \bar{Y} - Y_1$, $Y_2' = \bar{Y} - Y_2$

10. STANDARD FORMULA FOR CENTER OF GRAVITY

SHAPES	AREA	CENTER OF GRAVITY	
		\bar{X}	\bar{Y}
Rectangle	$b \times h$	$b/2$	$h/2$
Triangle	$1/2 \times b \times h$	$2/3 \times b$	$1/3 \times h$
Circle	$\pi d^2/4$	$d/2$	$d/2$
Semicircle	$\pi d^2/8$	$d/2$	$2d/3\pi$
Quarter circle	$\pi d^2/16$	$4r/3$	$4r/3\pi$

Where

- b= base length
- h=height of the section
- d=diameter
- r = radius

11. Center of gravity for compound section:

The center of gravity of compound sections formed by various common shapes of areas and lines may be found by splitting the compound sections into rectangles, triangles or other shapes.

$$\bar{X} = (A_1 X_1 + A_2 X_2 + A_3 X_3 \dots) / (A_1 + A_2 + A_3 \dots)$$

$$\bar{Y} = (A_1 Y_1 + A_2 Y_2 + A_3 Y_3 \dots) / (A_1 + A_2 + A_3 \dots)$$

X_1, X_2, X_3, \dots and Y_1, Y_2, Y_3, \dots are \bar{X} and \bar{Y} values of individual section. A_1, A_2, A_3, \dots are the areas of individual sections.

12. MI of composite area:

$$I_{XX} = \sum [(I_{CG})_{xx} + \text{Area} \times (\text{distance})^2]$$

$$I_{YY} = \sum [(I_{CG})_{yy} + \text{Area} \times (\text{distance})^2]$$

For I_{XX} , distance $h = (\bar{y} - y)$

I_{YY} , distance $h = (\bar{x} - x)$

13. SELF MOMENT OF INERTIA :
SHAPES SELF MOMENT OF INERTIA

SHAPES	SELF MOMENT OF INERTIA
Rectangle	$(I_{CG})_{XX} = bh^3/12$ $(I_{CG})_{YY} = hb^3/12$
Triangle	$(I_{CG})_{XX} = bh^3/36$ $(I_{CG})_{YY} = hb^3/48$
Circle	$(I_{CG})_{XX} = \pi d^4/64$ $(I_{CG})_{YY} = \pi d^4/64$
Semicircle	$(I_{CG})_{XX} = 0.0068D^4$ $(I_{CG})_{YY} = \pi d^4/128$
Quarter circle	$(I_{CG})_{XX} = 0.055 \cdot r^4$ $(I_{CG})_{YY} = 0.055 \cdot r^4$

14. PRINCIPAL MOMENT OF INERTIA:

$$\tan 2\theta = -2I_{XY} / (I_{XX} - I_{YY})$$

$$I_{max/min} = (I_{XX} + I_{YY})/2 \pm \sqrt{((I_{XX} - I_{YY})^2/2) + I_{XY}^2}$$

15. **The radius of gyration (r)** may be defined as the distance at which the entire mass of the body should be concentrated without altering the mass moment of inertia.

16. The methods of determining the centre of gravity..

1. By geometry
2. by method of moment
3. by integration

4. by graphical method.
17. the first moment of area
The term $\int x \, dA$ is known as the first moment of the area with respect to y- axis.
18. What are principal axes?
The axes about the moment of inertia are maximum and minimum are known as principal axes. The axes about the moment of inertia are maximum and minimum are known as principal axes and the corresponding inertia is principal moment of inertia.
19. $I_{xy} = 0$ for a figure which is symmetrical about either x or y axis.

MODULE IV - DYNAMICS OF PARTICLES

1. DISPLACEMENT (S)

It is the distance travelled by the particle. It is a scalar quantity. Unit:
meter

2. VELOCITY (V)

It is the rate of change of displacement. It is a vector quantity. Unit: ms⁻¹

$$V = ds/dt$$

3. ACCELERATION (a)

It is the rate of change of velocity. It is a vector quantity. Unit: ms⁻²

$$a = dv/dt = d^2s/dt^2$$

dt dt

Acceleration = Change of velocity = V-U

Time taken t

4. RETARDATION / DECELERATION

If initial velocity > final velocity then the acceleration is negative

5. SPEED

Speed = distance travelled

Time taken

6. AVERAGE SPEED

Average speed = total distance travelled

Total time taken

7. AVERAGE VELOCITY

Average velocity = (initial + final) velocity

2

$$= \frac{u+v}{2}$$

2

8. DISTANCE TRAVELLED (S)

s = velocity x time

$$s = vt$$

9. EQUATIONS OF MOTION IN A STRAIGHT LINE

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

u = 0, body starts from rest, v = 0, body comes to rest

$$10.1 \text{ Km/Hr} = 0.277 \text{ m/s} = \frac{1000 \text{ m}}{3600 \text{ s}}$$

3600

12. DISTANCE TRAVELLED IN N SECONDS

Distance travelled in nth second = S_n - S_{n-1}

$$S_n > S_{n-1}$$

$$S_{nth} = u + a(2n - 1)$$

2

13. EQUATION OF MOTION OF PARTICLE UNDER GRAVITY

when particle comes down:

$$h = ut + gt^2$$

$$v = u + gt$$

$$v^2 = u^2 + 2gh$$

when particle moves upwards :

$$h = ut - gt^2$$

$$v = u - gt$$

$$v^2 = u^2 - 2gh$$

14. MAXIMUM HEIGHT ATTAINED BY PROJECTED PARTICLE

$$(V=0) h_{\max} = \frac{u^2}{2g}$$

$$2g$$

15. TIME TAKEN TO REACH MAX. HEIGHT

$$(V = 0) t = \frac{u}{g}$$

$$g$$

16. TOTAL TIME TAKEN

Total time taken = 2 x time taken to reach max. Height

$$= \frac{2u}{g}$$

$$g$$

17. STRIKING VELOCITY OF THE PARTICLE

$$(U = 0) v = (2gh)^{1/2}$$

Velocity with which a particle is thrown ($v = 0$)

$$u = (2gh)^{1/2}$$

18. DISPLACEMENT IN TERMS OF TIME

$$s = \int v dt$$

$$v = \frac{ds}{dt} = \frac{d}{dt} f(t)$$

$$dt dt$$

$$a = \frac{dv}{dt} = \frac{d^2}{dt^2} f(t)$$

$$dt dt^2$$

19. EQUATION OF MAXIMUM VELOCITY

Equation of maximum velocity $\frac{dv}{dt} = 0$

$$dt$$

20. RECTILINEAR MOTION WITH VARIABLE ACCELERATION

$$a = f(t)$$

$$\int a dt = \int f(t) dt$$

$$s = \int \int f(t) dt dt$$

$$= \int v dt$$

$$a = v \frac{dv}{ds}$$

$$ds$$

21. RELATIVE VELOCITY: $V_{B/A} = V_B - V_A$ Here $V_B > V_A$

$$\int V_{A/B} = V_A - V_B \text{ Here } V_A > V_B$$

$$\int V_{B/A} = (V_B^2 + V_A^2)^{1/2}$$

$$\int \square = \tan^{-1} \frac{V_B}{V_A}$$

$$V_A$$

22. RELATIVE DISPLACEMENT:

$$S_{B/A} = S_B - S_A$$

$$S_{B/A} = (S_A^2 + S_B^2)^{1/2}$$

$$\square = \tan^{-1} \frac{S_B}{S_A}$$

$$S_A$$

23. RELATIVE ACCELERATION:

$$a_{B/A} = (a_A^2 + a_B^2)^{1/2}$$

$$\square = \tan^{-1} \frac{a_B}{a_A}$$

aA

24. KINEMATICS OF PARTICLES – CURVILINEAR MOTION:

VELOCITY:

Horizontal component, $V_x = \frac{dx}{dt}$

dt

Vertical component, $V_y = \frac{dy}{dt}$

dt

Resultant velocity, $v = (v_x^2 + v_y^2)^{1/2}$

Angle of inclination with x – axis $\theta = \tan^{-1} \frac{v_y}{v_x}$

v_x

25. ACCELERATION: $a = (a_x^2 + a_y^2)^{1/2}$

$\theta = \tan^{-1} \frac{a_y}{a_x}$

a_y

a_x

Horizontal Component:

$a_x = \frac{dv_x}{dt}$

dt

Vertical Component:

$a_y = \frac{dv_y}{dt}$

dt

26. NORMAL ACCELERATION:

$a_n = \frac{v^2}{r}$

r

v = velocity of the particle

r = radius of the curvature of curvilinear motion

27. RESULTANT ACCELERATION:

$a = (a_t^2 + a_n^2)^{1/2}$

a_t = tangential acceleration

a_n = normal acceleration

POLAR SYSTEM (r, θ)

28. VELOCITY COMPONENT: radial component $V_r = \frac{dr}{dt}$

dt

Transverse component $V_\theta = r \frac{d\theta}{dt}$

dt

$V = (V_r^2 + V_\theta^2)^{1/2}$

$\theta = \tan^{-1} \frac{V_\theta}{V_r}$

V_θ

29. ACCELERATION COMPONENT:

Radial component $a_r = \frac{d^2 r}{dt^2} - r \left(\frac{d\theta}{dt}\right)^2$

$dt^2 dt$

Transverse component $a_\theta = r \frac{d^2 \theta}{dt^2} + 2 \frac{dr}{dt} \frac{d\theta}{dt}$

$dt dt dt$

30. PROJECTILE: A Particle projected in the space at an angle to horizontal. Plane is called projected. And motion is called projectile motion.

31. TIME OF FLIGHT: $t = \frac{2u \sin \theta}{g}$

g

$u \sin \theta$ = initial velocity in the upward (vertical component)

θ = angle of projection

total time $T = \frac{2u \sin \theta}{g}$

g

32. MAXIMUM HEIGHT ATTAINED:

$h_{\max} = \frac{u^2 \sin^2 \theta}{2g}$

$2g$

33. RANGE, (R) $R = \frac{u^2 \sin 2\theta}{g}$

Maximum horizontal range, $R = \frac{u^2}{g}$

g

when $\theta = 45^\circ$

34. VELOCITY AND DIRECTION OF THE PROJECTILE

$V_x = u \cos \theta$

$V_y = (u \sin \theta)^2 - 2gh$

$V = \sqrt{V_x^2 + V_y^2}$

35. MOTION OF PARTICLE FROM KNOWN HEIGHT

Range = Horizontal Velocity \times time taken

$= u \times t$

$h = \frac{gt^2}{2}$

2

36. WORK DONE

Work Done = Force \times displacement

Unit = Nm = Joules

37. POWER

Rate of doing work

Power = $\frac{\text{Work done}}{\text{Time}}$

Time

= $\frac{\text{Force} \times \text{displacement}}{\text{Time}}$

Time

= $\frac{\text{force} \times \text{velocity}}{\text{Time}}$

= unit: Nms^{-1} = watt

NEWTONS LAWS OF MOTION

38. MOMENTUM Momentum = Mass \times Velocity

$M = mv$

Unit: Kgm^{-1}

1 Newton = kgm^{-1}

39. WEIGHT

Weight = mass \times acceleration due to gravity

$W = mg$

Unit: Newton

40. STATIC EQUATION OF EQUILIBRIUM

$\sum H = 0$; $\sum V = 0$; $\sum M = 0$

41. D'ALEMBERTS EQUATIONS OF EQUILIBRIUM

$\sum P - ma = 0$

$a = \frac{P}{m}$

m

$$a = P_1 + P_2 + P_3 \dots\dots\dots$$

m

D'Alembert's principle states that a system of forces acting on a body in motion is in equilibrium with the inertia force or imaginary force (ma) of the body

42. MECHANICAL ENERGY: P.E = $mg \times h$

$$= (\text{Force}) \times (\text{Displacement})$$

43. KINETIC ENERGY:

$$KE = \frac{1}{2}mv^2 \text{ [if } u = 0]$$

$$= \frac{1}{2}m(v^2 - u^2) \text{ [if } u \neq 0]$$

$$= w(v^2 - u^2)$$

2g

$$= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \text{ Work done by body in motion} = \text{final kinetic energy} - \text{initial kinetic energy}$$

$\sum F_x$ = sum of forces that induce the motion of a body.

44. WORK DONE BY SPRING BODY:

$$\text{Work done} = -\frac{1}{2}kx^2$$

Unit: N/mm = KN/m

K = spring constant / spring modulus (depends upon the material used)

$K = f$, x = maximum deformation,

x

$$f = kx$$

45. DEFORMATION TO NORMAL POSITION:

$$\text{Work done} = \frac{1}{2}kx^2$$

Unit: N/mm

46. WORK DONE BY A SYSTEM:

= work done by the block + work done by the spring

47. SPRING CONSTANT:

= load.

Deflection

Unit: N/m

IMPULSE – MOMENTUM

48. IMPULSE OF A FORCE

When a large force acts for a short period of that force is called an impulsive force. It is a vector quantity

$$\text{Impulse (I)} = \text{force} \times \text{time}$$

Unit = Ns

49. MOMENTUM Momentum = mass \times velocity

$$M = mv$$

Unit = Kgms⁻¹

50. IMPULSE-MOMENTUM EQUATION

$$\text{Impulse} = \text{Final momentum} - \text{Initial momentum}$$

51. LAW OF CONSERVATION OF LINEAR MOMENTUM

$$\text{Initial momentum} = \text{Final momentum}$$

Total momentum before impact = Total momentum after impact

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

52. IMPACT An sudden short term action between two bodies

53. ELASTIC BODY If body retains its original shape and size when the external forces are removed, then the body is said to be perfectly elastic body.

54. PERIOD OF DEFORMATION

The time elapsed from the instant of initial contact to the maximum deformation is period of deformation.

55. PERIOD OF RESTITUTION

The time elapsed from the instant of maximum deformation to the instant of just separation is called period of restitution.

period of impact = period of deformation + period of restitution

56. NEWTON'S LAW OF COLLISION (CO-EFFICIENT OF RESTITUTION)

Coefficient of restitution (e) = Impulse during restitution

Impulse during deformation

$$= V_2 - V_1$$

$$u_1 - u_2$$

57. IMPACT OF ELASTIC BODY WITH A FIXED PLANE

Coefficient of restitution = Velocity of restitution

Velocity of deformation

$$e = - \frac{(2gh)^{1/2}}{(2gH)^{1/2}}$$

$$h = e^2H$$

58. LINE OF IMPACT

It is an imaginary line passing through the point of contact and normal to the plane of contact

59. DIRECT IMPACT The velocities of two colliding bodies before impact and after impact are collinear with the line of impact

60. OBLIQUE IMPACT The velocities of two colliding bodies after collision are not collinear with the line of impact

Vertical component before impact = vertical component after impact

Horizontal component before impact = horizontal component after impact

MODULE V

FRICITION AND ELEMENTS OF RIGID BODY DYNAMICS

1) Friction The tangential force developed in opposite direction of the movement of object is called force of friction or frictional force or simply friction.

Types of friction Dry friction _absence of lubricating oil.

i. Static friction _friction of body under rest.

ii. Dynamic (Kinetic) friction _friction of body under motion.

2. Fluid friction _presence of thin film of fluid.

2) Limiting friction

The maximum resistance offered by a body against the external force which tends to move the body is called limiting force of friction.

3) Coefficient of friction (μ)

$$\mu = \frac{F_m}{N_R}$$

N_R

F_m _ Force of friction.

N_R _ Normal reaction.

4) For static friction

$$(F_m)_s = \mu_s N_R > (F_m)_k$$

For dynamic friction

$$(F_m)_k = \mu_k N_R$$

5) Reaction of friction

$$R = \mu (N_R + F_m)$$

$2 + F_m$

2)

6) Angle of friction (α)

α _ angle between normal reaction and reaction is called angle of friction.

$$\tan \alpha = \frac{F_m}{N_R}$$

N_R

7) If a body is under equilibrium under the action of W & N_R

Case (i) $F=0$, $W=N_R$

Case (ii) $F < F_m$, $W=N_R$, $F=F_m$

Case (iii) $F=F_m$, $W=N_R$, $F_m = \mu N_R$

Case (iv) $F > F_m$ (body gets movement) , $W=N_R$, $F=F_m$, $F_m = \mu N_R$

8) Impending motion

The state of motion of a body which is just about to move or slide is called impending motion.

9) Coulomb's law of static friction

_ F_m opposite to direction of P (applied force)

_ F_m independent of shape of body and area of contact

_ F_m depends on degree of roughness of surface

_ $F_m = P$ when body is at rest

_ $(F_m)_s = \mu_s N_R$

10) Coulomb's law of dynamic or kinetic friction

_ F_m opposite to direction of P

_ $(F_m)_k < (F_m)_s$

_ $\mu_k < \mu_s$

_ $(F_m)_k = \mu_k N_R$

11) Angle of repose (α_m)

Angle of inclined plane at which the body tends to slide down is known as angle of repose.

$$\mu = \tan \alpha_m = \tan \alpha$$

$\mu_m = \mu$ at the time of repose.

12) Simple contact friction

The type of friction between the surface of block and plane is called simple contact friction.

13) Applications of simple contact friction

_ Ladder friction

_ Screw friction

_ Belt friction

14) Screw jack

Screw jack is a device used to lift or lower loads gradually.

15) Screw friction

Friction in screw jack.

16) Force applied at lever

_ While lifting $P_1 = Wr \tan(\alpha + \phi)$

L

_ While lowering $P_1 = Wr \tan(\alpha - \phi)$

L

17) Pitch Pitch (p) is the distance moved by the screw in one cycle rotation.

18) Statically determinate structure

If a body or structure can be analysed using equilibrium

equations $\sum H=0$, $\sum V=0$, $\sum M=0$ then it is said to be statically determinate structure.

19) Force transmitted on screw

To lift

$P = W \tan(\alpha + \phi)$

To lower

$P = W \tan(\alpha - \phi)$

helical angle

angle of friction

20) Screw friction

Helical angle $\alpha = \tan^{-1} p$

d

Angle of friction $\phi = \tan^{-1} \frac{F_m}{N_R}$

N_R

p pitch

d diameter F_m Friction force

N_R Normal reaction

21) Belt friction

T_2

$= e^{\mu \theta}$

T_1

T_1 Tension in slack side

T_2 Tension in tight side

Coefficient of friction between belt and wheel

Angle of contact

22) Power transmitted

$P = (T_2 - T_1) \times V$ Nms watt

V velocity of belt

23) Torque on driver $= (T_2 - T_1) \times r$ N-m

Torque on follower = $(T_2 - T_1) \times r$ N-m

Rolling resistance

When a body is made to roll over another body the resistance developed in the opposite direction of motion is called the rolling resistance. $R \cos \phi$ is called rolling resistance.

24) Coefficient of resistance

Horizontal distance b is called as coefficient of resistance.

$b = Pr$

W

P applied force

r radius of the body

W weight of the body

25) Types of motion of rigid bodies

_ Translation

a) Rectilinear straight line path motion

b) Curvilinear curved path motion

_ Rotation (with respect to a fixed point)

_ Rotation & Translation (General plane method)

27) Angular displacement (θ)

Total angle which a body has rotated in t seconds.

$\theta = \int \omega dt$

28) Angular velocity (ω)

Rate of change of angular displacement.

$\omega = \frac{d\theta}{dt}$ rads

dt

29) Angular acceleration (α)

Rate of change of angular velocity.

$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$ rad/s²

$dt dt^2$

Equations of Angular motion

_ $\theta = \omega_0 t + \frac{1}{2} \alpha t^2$

2

_ $\omega = \omega_0 + \alpha t$

_ $\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$

$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

31) Relation between linear and angular motion

$s = r\theta$

$v = r\omega$

$a = r\alpha$

32) If angular velocity (ω) is uniform then angular acceleration

(α) = 0

Hence $\theta = \omega t$

33) Angular velocity (ω) is represented as rad/s or rev/min (rpm)

$\omega = \frac{2\pi N}{60}$

60

N rpm of rotating body.